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A welcome from Organizers

Dear participant, it is our pleasure to welcome you to the first Symposia on Mathematical Techniques Applied to Data Analysis and Processing (SMATAD) at Fuengirola (Málaga). We offer you four little symposiums devoted to four different topics: *Contradiction and lack of information in knowledge data bases, Formal Concept Analysis, Fuzzy techniques in Image Processing* and *Forecasting and optimization for data-driven decision-making under uncertainty.* We hope to provide, with this event, a forum for a fruitful interaction and exchange of knowledges among those four different areas of applied mathematics

There is a better interchange of ideas with a good social environment. For that reason, besides the technical program, we have prepared for Friday a social trip and we invite you to enjoy evenings with the rest of participants.

We wish you to have a good time during SMATAD.

Nicolás Madrid

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Plenary talks

15:00-16:00 Thursday 18th

Ambiguity and Hesitancy in Quality Assesment: The case of Image segmentation

Carlos Lopez-Molina (Universidad Pública de Navarra, SPAIN)

Automatic information processing regularly deals with plenty of sources of uncertainty. Many of them are born from the data gathering itself, while many others stem from imprecise computations or algorithmic needs. Human beings generally make variable interpretation of the goals and needs of an information processing task, whichever context it is carried out in. Hence, the perceived quality of one single result will be heterogeneous, depending on the human expert evaluating it. This poses significant problems in various stages of information processing. For example, it is damaging when it comes to algorithmic setting or optimization, since the perceived improvement (for one human) might be coupled to the perceived quality loss (according to another human). Also, it becomes damaging when scientists intend to select the best performing algorithm for a given task, as the opinions by different human experts might differ. In general, we find that the perceived quality of one single result is a conglomerate of opinions, often hesitant or contradictory.

The problem of ambiguous data for quality assessment is common in Image Processing. One clear example is the task of Image segmentation, which lacks a mathematical definition, and hence automatic methods are bound to be evaluated according to how similar their results are to human-made solutions. Unfortunately, different humans routinely produce different interpretations of an image. As a consequence, the evaluation of a segmented image becomes some sort of comparison with a list of human-made, spatially imprecise segmentations. This requires a significant mathematical apparatus which is able to cope with multivariate data involving hesitancy, ambiguity and contradiction.

In this talk we analyze, from a historical perspective, the problem of quality evaluation for image segmentation. Specifically, we focus on how to handle the variable interpretation by different humans. This will lead to an analysis of the general quality evaluation problem in the presence of multivariate ground truth, including its semantics, the technical challenges it poses and the relationship with some mathematical disciplines involved in its solution.

C. Lopez-Molina received the Ph.D. degree from the Universidad Publica de Navarra, in 2012, where he is currently an Assistant Professor. His research interests are in low-level feature extraction/treatment for computer vision and automated bioimagery processing. He has developed most of his work around edge and boundary detection, and in soft computing techniques for computer vision.

10:00-11:00 Saturday 20th

Data-Driven Distributionally Robust Optimization Using the Wasserstein Metric

Daniel Kuhn (Risk Analytics and Optimization at EPFL,Switzerland)

We consider stochastic programs where the distribution of the uncertain parameters is only observable through a finite training dataset. Using the Wasserstein metric, we construct a ball in the space of probability distributions centered at the uniform distribution on the training samples, and we seek decisions that perform best in view of the worst-case distribution within this ball. We show that the resulting optimization problems can be solved efficiently and that their solutions enjoy powerful out-of-sample performance guarantees on test data. The wide applicability of this approach is illustrated with examples in portfolio selection, uncertainty quantification, statistical learning and inverse optimization.

Daniel Kuhn holds the Chair of Risk Analytics and Optimization at EPFL. Before joining EPFL, he was a faculty member at Imperial College London (2007-2013) and a postdoctoral researcher at Stanford University (2005-2006). He received a PhD in Economics from the University of St. Gallen in 2004 and an MSc in Theoretical Physics from ETH Zurich in 1999. His research interests revolve around robust optimization and stochastic programming.

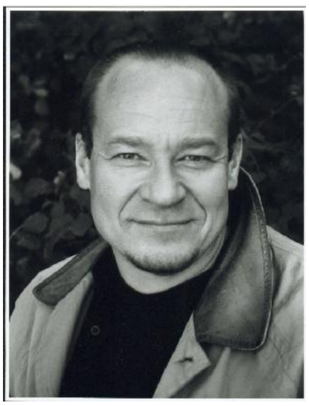
10:00-11:00 Sunday 21th

Connecting paraconsistent and many-valued logic in decision making task

Esko Turunen (Tampere University of Technology, Finland)

Paraconsistent logic refers to non-classical systems of logic, which reject the Principle of explosion; once a contradiction has been asserted, any proposition can be inferred. The primary motivation for paraconsistent logic is the conviction that it ought to be possible to reason with inconsistent information in a controlled and discriminating way. The principle of explosion precludes this, and so must be abandoned. The key note is evidence; if a statement is true, then there is also some evidence in favor of it, while the converse does not necessarily hold. If there is evidence in favor of a statement, it need not be true; there may also be evidence against it. Paraconsistent logics make it possible to reason with inconsistent information. Many-valued or fuzzy logics in turn are logic calculus in which there are more than two truth-values. Classical two-valued logic may be extended to infinite-valued Łukasiewicz logic and its extension, called Pavelka logic, has infinitely many truth-values and provability degrees. We show how paraconsistency and many-valuedness can be combined in Pavelka's logic framework. Moreover, we show a real life application in decision-making how this logic system can be utilized.

Professor Esko Turunen earned his PhD in applied mathematics, entitled 'A Mathematical Study of Fuzzy Logic: an Algebraic Approach' in 1994 at Lappeenranta University of Technology, Finland. At present Turunen is the head of the Department of Mathematics at Tampere University of Technology, Finland. His scientific interests and research areas are in many-valued logics, fuzzy logic, paraconsistent logic, data mining and their real life applications. Turunen has published more than 50 scientific articles and conference papers with peer review process including three books. Turunen has wide international contacts to several universities all over the world; he has spent more than ten years in various universities and research institutes including Charles University and Prague University of Technology in Czech Republic, University of Naples, Salerno and Pisa in Italy and Technical University in Vienna, Austria. Turunen has supervised several doctoral theses and is a member of many editorial boards of scientific journals. Turunen has represented Finland in three COST Action research projects and is currently the representative of his country in COST Action IC1406. In addition to theoretical mathematical research, Turunen has worked as a mathematical expert in many industrial research projects, such as a developer of intelligent traffic systems, medical expert systems designer and creator of various other control systems.



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Connecting paraconsistent and many-valued logic in decision making task
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Contradiction and lack of information in knowledge-data-bases I (11:30-13:00)

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Characterizing the existence of residual for a morphism between a fuzzy preposet and an unordered fuzzy structure

I. P. Cabrera, and P. Cordero, and M. Ojeda-Aciego

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Since their inception, Galois connections have shown to be an interesting tool both for theory and for applications [1,2]. One particular research area on application is Formal Concept Analysis (FCA) [1], since the main properties of the concept-forming operators are consequences of their being part of a Galois connection, both in crisp and fuzzy cases.

In the recent years, we have studied the problem of constructing the right adjoint of a mapping $f\colon A \to B$ between *differently structured* environments, in the sense that A has a richer structure than B. Firstly, the missing structure on B has to be built and then, the right adjoint should be defined.

A number of results have been obtained so far on different underlying settings. Namely, in [4] we worked with crisp functions between a poset (resp. preordered set) and an unstructured set; later, in [5] we entered in the fuzzy arena, considering the case in which A is fuzzy preposet; then, in [6], we extended the previous results by allowing fuzzy equivalence relations as an adequate substitute to equality.

More recently, in [7], we started to find a more adequate definition of adjunction in a fuzzy environment, since the fuzzy extensions given in [5, 6] lack of fuzziness precisely on the adjunction, i.e. both components of the adjunction are crisp, and we would like to work with a really fuzzy version of the notion of adjunction, in which they are fuzzy functions as well.

Our aim now is an alternative generalization in which the components of the fuzzy adjunction are fuzzy relations satisfying less requirements than a fuzzy function. The construction should embed previous notions of fuzzy Galois connection as a particular case but it must represent a real progress in the theory.

We are planning to continue the line initiated in [5, 6] and attempt the construction of the residual, in the sense of relational fuzzy Galois connections, to a given mapping between differently structured domain and codomain.

References

[1] K. Denecke, M. Erné, and S. Wismath, *Galois connections and applications*, Mathematics and its Applications. Springer, 2004, vol. 565.

[2] F. García-Pardo, I.P. Cabrera, P. Cordero, and M. Ojeda Aciego, "On Galois connections and soft computing," *Lect. Notes in Computer Science*, vol. 7903, pp. 224–235, 2013.

[3] W. Yao and L.-X. Lu. Fuzzy Galois connections on fuzzy posets. *Mathematical Logic Quarterly (1)* 55 (2009), 105–112.

Acknowledgement The support of the Spanish MINECO project RASO (TIN2015-71799-C2-1-P is kindly acknowledged.

[4] F. García-Pardo, I.P. Cabrera, P. Cordero, M. Ojeda Aciego, and F. Rodríguez, "On the definition of suitable orderings to generate adjunctions over an unstructured codomain," *Information Sciences*, vol. 286, pp. 173–187, 2014.

[5] I.P. Cabrera, P. Cordero, F. García-Pardo, M. Ojeda-Aciego, and B. De Baets, "On the construction of adjunctions between a fuzzy preposet and an unstructured set," *Fuzzy Sets and Systems* 2017. To appear. http://dx.doi.org/10.1016/j.fss.2016.09.013

[6] ——, "Adjunctions between a fuzzy preposet and an unstructured set with underlying fuzzy equivalence relations," 2017, Submitted.

[7] I.P. Cabrera, P. Cordero, and M. Ojeda-Aciego, "On fuzzy relations, functional relations, and adjunctions," *Proc. of Foundations of Computational Intelligence* 2016. https://doi.org/10.1109/SSCI.2016.7850149

[8] B. Ganter and R. Wille, *Formal Concept Analysis: Mathematical Foundation.* Springer Verlag, 1999.

Fuzzy Associative Memory in the Context of Formal Concept Analysis

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One of the first publications devoted to *fuzzy associative memories* (FAM) has been made by Kosko - [1]. The FAM has been characterized as a single-layer feedforward neural net performing a nonlinear matrix-vector multiplication. This approach was later extended with the purpose to increase the storage capacity. Significant progress was achieved by introduction of learning implication rules, that afterwards led to *implicative fuzzy associative memory* (IFAM) with *implicative fuzzy learning*. A justification of validity of a certain IFAM model was discussed in [4] where the characterization of one type of suppressed noise - eroded - was proposed.

In the current contribution, we use the framework of formal concept analysis (FCA) and propose the theory of FAM in the language of FMA. Let us remark that FCA is successively used in e.g., fuzzy property-oriented concept lattices, and in a number of applications such as modeling and processing of incomplete knowledge in information systems [2].

We formalize two principal activities of FAM: data retrieval and noise reduction. We use the proposed formalism and show that the problem of data retrieval is connected with solvability and eigen sets of a certain system of fuzzy relation equations. We differentiate FAM models according to their ability to reduce noise and show how the choice of formal context determines a type of noise that can be reduced by the corresponding retrieval mechanism.

From the technical point of view, we extend the theory of FAM by using a general algebraic structure instead of a specific one (Łukasiewicz algebra in [4] and by enlarging the set of autoregressive fuzzy associative memory models (AFAM). We propose a formal characterization of AFAM models and show the way of various modifications, see also [3]. We analyze the retrieval mechanism of AFAM and its ability to remove noise. We show that the larger is the amount of noise, the greater should be the fuzzy relation that models retrieval with noise reduction. Further, we show how the type of removable noise depends on which type of AFAM models is applied. Finally, we construct a fast algorithm of data retrieval and give illustration of the noise reduction.

We choose and fix a finite set X and a complete residuated lattice $\mathbf{L}$. The following *formal context* $\mathcal{K} = (\mathcal{A}, \mathcal{R}, \mathcal{I}_{\boxdot})$ where $\mathcal{A} \subseteq L^X$ is a set of objects, $\mathcal{R} \subseteq L^{X\times X}$ is a set of attributes, and $\mathcal{I}_{\boxdot}$ is an incidence relation on $\mathcal{A} \times \mathcal{R}$, is proposed. We say that an object (dataset) A possesses an attribute R, if the latter is an AFAM model of A via the composition $\boxdot$. Equivalently, $\mathcal{I}_{\boxdot}(A, R) = 1$ if and only if $A \in \mathcal{A}$ is an eigen fuzzy set of $R \in \mathcal{R}$ with respect to the composition $\boxdot$. A couple $(\mathbf{A}, \mathbf{W})$ is a *formal concept* of $\mathcal{K}$, if $\mathbf{A} \subseteq \mathcal{A}$ is a dataset, $\mathbf{W} \subseteq \mathcal{R}$ is a set of fuzzy relations such that $A \in \mathbf{A}$ if and only if A is an eigen fuzzy set of any $W \in \mathbf{W}$

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with respect to the composition $\boxdot$, and vise versa, $W \in \mathbf{W}$ if and only if $(W, \boxdot)$ is an AFAM model of $\mathbf{A}$. Using both languages, we say that every element in dataset $\mathbf{A}$ has every attribute in $\mathbf{W}$, i.e. can be successfully retrieved from its sample using any AFAM model $(W, \boxdot)$ where $W \in \mathbf{W}$ and composition $\boxdot$ is clear from the context $\mathcal{K}$. We characterize formal concepts where datasets as formal concept objects are connected with retrieval models as formal concept attributes.

We differentiate FAM models according to their ability to reduce noise and showe how the choice of formal context determines a type of noise that can be reduced by the corresponding retrieval mechanism.

References

[1] Kosko, B.: *Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence.* Prentice-Hall. (1992).

[2] Alcalde, C., Burusco, A., Daz-Moreno, J. C., Fuentes-Gonzlez, R., Medina, J.: *Fuzzy property-oriented concept lattices in morphological image and signal processing.* Lect. Notes in Computer Science. **7903** (2013) 246 - 253.

[3] Perfilieva, I., Vajgl, M.: *Autoassociative Fuzzy Implicative Memory on the Platform of Fuzzy Preorder.* Proc. of IFSA-EUSFLAT 2015, Atlantis Press, 2015.

[4] Sussner, P. , Valle, M.-E.: *Implicative Fuzzy Associative Memories.* IEEE Trans. Fuzzy Systems **14** (2006) 793–807.

Intuitionistic L-fuzzy Formal Concept Analysis

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FCA arose some thirty years ago as an applied lattice theory from the seminal work by Ganter and Wille [1]. Originally, the theory was developed in the crisp but was soon extended to the fuzzy case by Burusco and Fuentes-González [2] and, since then, a great number of different generalizations have been developed: by Bělohlávek using a complete residuated lattice [3], the (one-sided) generalized FCA by Krajči [4], multi-adjoint FCA by Medina et al [5–8], heterogeneous and higher-order FCA by Krídlo et al [9, 10], etc.

Our approach here will be based on the intuitionistic fuzzy sets (IF-sets for short) introduced by Atanassov [11]. His original construction was later adapted to the L-fuzzy case, in which a complete residuated lattice was used instead of the unit interval as underlying set of truth-values [12, 13]. by considering for all element x a membership degree $\mu(x)$ together with a non-membership degree $\nu(x)$ such that $\mu(x) + \nu(x) \leq 1$, somehow allowing an *indetermination degree* about x in the case of strict inequality.

Although some authors have already introduced IF-based ideas within the FCA framework: for instance, [15, 16] define a generalization based on Krajči's one-sided approach; on the other hand, [17] focuses on an interval-valued intuitionistic fuzzy rough approach. And all three previous approaches are based on the unit interval.

We introduce an adjoint triple of operators defined on an IF complete residuated lattice of truth-values which, naturally, allows for introducing the notion of IF-L-fuzzy formal t-concept.

References

[1] B. Ganter and R. Wille, *Formal Concept Analysis: Mathematical Foundation.* Springer Verlag, 1999.

[2] A. Burusco and R. Fuentes-González, "The study of L-fuzzy concept lattice," *Mathware & Soft Computing*, vol. 3, pp. 209–218, 1994.

[3] R. Bělohlávek, "Lattice generated by binary fuzzy relations (extended abstract)," in *4th Intl Conf on Fuzzy Sets Theory and Applications*, 1998, p. 11.

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Manuel Ojeda-Aciego is partially supported by the Spanish Science Ministry project TIN15-70266-C2-P-1, co-funded by the European Regional Development Fund (ERDF).

[4] S. Krajči, "The basic theorem on generalized concept lattice," in *International Workshop on Concept Lattices and their Applications, CLA 2004*, V. Snásel and R. Bělohlávek, Eds., 2004, pp. 25–33.

[5] J. Medina, M. Ojeda-Aciego, and J. Ruiz-Calviño, "Formal concept analysis via multi-adjoint concept lattices," *Fuzzy Sets and Systems*, vol. 160, no. 2, pp. 130–144, 2009.

[6] J. Medina and M. Ojeda-Aciego, "Multi-adjoint t-concept lattices." *Information Sciences*, vol. 180, pp. 712–725, 2010.

[7] ——, "Dual multi-adjoint concept lattices," *Information Sciences*, vol. 225, pp. 47 – 54, 2013.

[8] J. Konecny, J. Medina, and M. Ojeda-Aciego, "Multi-adjoint concept lattices with heterogeneous conjunctors and hedges," *Annals of Mathematics and Artificial Intelligence*, vol. 72, pp. 73–89, 2014.

[9] L. Antoni, S. Krajči, O. Krídlo, B. Macek, and L. Pisková, "On heterogeneous formal contexts," *Fuzzy Sets and Systems*, vol. 234, pp. 22–33, 2014.

[10] O. Krídlo, S. Krajči, and L. Antoni, "Formal concept analysis of higher order," *Int. J. General Systems*, vol. 45, no. 2, pp. 116–134, 2016.

[11] K. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, pp. 87–96, 1986.

[12] ——, *Intuitionistic Fuzzy Sets: Theory and Applications.* Physica- Verlag, 1999.

[13] T. Gerstenkorn and A. Tepavčević, "Lattice valued intuitionistic fuzzy sets," *Central European Journal of Mathematics*, vol. 2, no. 3, pp. 388–398, 2004.

[14] O. Krídlo and M. Ojeda-Aciego, "Extending Formal Concept Analysis using intuitionistic L-fuzzy sets," Univ Málaga, Dept of Applied Mathematics, Tech Report 17-01, 2017.

[15] J. Pang, X. Zhang, and W. Xu, "Attribute reduction in intuitionistic fuzzy concept lattices," *Abstract and Applied Analysis*, 2013, article ID 271398. 12 pages.

[16] L. Zhou, "Formal concept analysis in intuitionistic fuzzy formal context," in *Seventh International Conference on Fuzzy Systems and Knowledge Discovery (FSKD 2010)*, 2010, pp. 2012–2015.

[17] F. Xu, Z.-Y. Xing, and H.-D. Yin, "Attribute reductions and concept lattices in interval-valued intuitionistic fuzzy rough set theory: Construction and properties," *Journal of Intelligent and Fuzzy Systems*, vol. 30, no. 2, pp. 1231–1242, 2016.

Simplification Logic as the tool to build efficiently direct bases in Formal Concept Analysis

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Formal Concept Analysis is a robust framework to store information, discover knowledge and manage it efficiently. There are some kinds of knowledge representation: concept lattices, closure systems and implications. In this work we focus on how to manipulate sets of implications using Simplification Logic. Specifically, we present the use of this tool to compute minimal sets of implications (bases) with an interesting property in Formal Concept Analysis: directness. A summary of the new methods developed to compute the direct-optimal basis, D-basis and DD-basis is showed.

Introduction and background

Formal Concept Analysis (FCA) is a useful tool for knowledge discovering from a dataset. FCA has been used in different areas: Artificial Intelligence, Databases, Software Engineering, Data Mining, and recently in the Semantic Web. For an introduction to FCA we refer the reader to [4].

We use the logic to deal with one of the most interesting ways to represent this hidden knowledge, that is, the set of *implications* (the implicational system - IS). Armstrong's Axioms [1] is the former axiomatic system introduced to manage ISs in a logical style, unfortunately, this axiomatic system is not suitable to develop automated methods. To this aim, in [5, 6] we have proved the efficiency of the use of Simplification Logic ($\mathbf{SL}_{FD}$ for short), which is an equivalent logic to Armstrong's Axioms.

Among the most interesting properties for ISs we emphasize minimality (basis) and *directness*. This last property is useful to compute the closure of a given set of attributes in only one traversal of the IS. A basis fulfilling this property is named *direct basis*.

Our main issue is the improvement of the computation of this kind of direct bases in the literature: the direct-optimal basis [2, 3] and the D-basis [ANR13]. As we have said, to reach this efficiency we use $\mathbf{SL}_{FD}$ as the kernel of the methods. Finally, a new direct basis is proposed: the DD-basis (dichotomous direct basis).

Computing Direct Bases

The time complexity of the methods for computing these direct bases from an arbitrary IS is exponential. This issue motivates the idea of obtaining other methods that, without avoiding the intrinsic exponential complexity of the problem, provides a better performance than previous works.

Acknowledgement This work has been partially supported by grant TIN2014-59471-P of the Science and Innovation Ministry of Spain.

In [2,3] the authors have proposed methods for the basis fulfilling the properties of optimality and directness: the direct-optimal basis. For this issue, we have used $\mathbf{SL}_{FD}$ [7, 9] as the core of the development of more efficient methods to compute it. In [7] we introduced a new method named *doSimp*. The inspiration of the method proposed in this paper is to maintain the intermediate ISs reduced at any time by means of the application of inference rules that always produce non-redundant implications. Basically, the method [7] has a Simplification stage (applying simplification equivalence of $\mathbf{SL}_{FD}$ [5]), a Completion stage (applying strong simplification rule [7]) and a Optimization stage (applying right-simplification equivalence of $\mathbf{SL}_{FD}$ [5]). In a first experiment [7] we compare *doSimp* with the previous methods. Although those methods do not finish when a 15-implicational system is received as an input, the new algorithm finishes in less than 10 seconds. In [9] an improvement of *doSimp*, named *SLgetdo*, has been proposed with the idea of using Simplification in all the algorithm interleaving the constantly application of the reduction paradigm. The left part of Figure 1 shows the significant results achieved.

The D-basis introduced in [ANR13] employs the concept of *ordered direct* computation of closures to find the way to shorten the direct-optimal basis. The D-basis is usually a proper subset of the direct-optimal basis and it preserves the property of computation of closures.It remained an open problem to develop an algorithm of generating the D-basis from an arbitrary implicational system until we designed an algorithm solving this issue: the `D-basis` algorithm. It takes into account the connection between *minimal generators*, essential components of a direct basis, and *minimal covers*, which are essential in the D-basis (see [8]). Later on, the next challenge was to approach how to integrate the computation of the minimal generators and minimal covers, exploiting the theoretical relationships between them searching for a more efficient method. We proposed a new algorithm `FastD-basis` [11] to compute the D-basis from an arbitrary set of implications Σ. The algorithm uses $\mathbf{SL}_{FD}$ to interweave the selection of the minimal covers in each step rather than in a final step. The central part of Figure 1 shows the comparison between `D-basis` and `FastD-basis` methods.

Finally, we face the definition of a new direct basis. In [10] we carry out a study of the set of implications, rendering a dichotomous partition of the whole set of implications according to their behavior with respect to the closure operator. The new basis definition, called dichotomous basis - DD-basis - is strongly based on the separate treatment of two kinds of implications: quasi-key implications and no quasi-key implications. The new approach reduces the size of the subset of implications withstanding the exponential cost of the basis construction process. This reduction comes from the removal of the quasi-key implications in the exponential task. Two methods have been proposed in [10] and the right part of Figure 1 shows the comparison between them and our method to compute the direct-optimal basis.

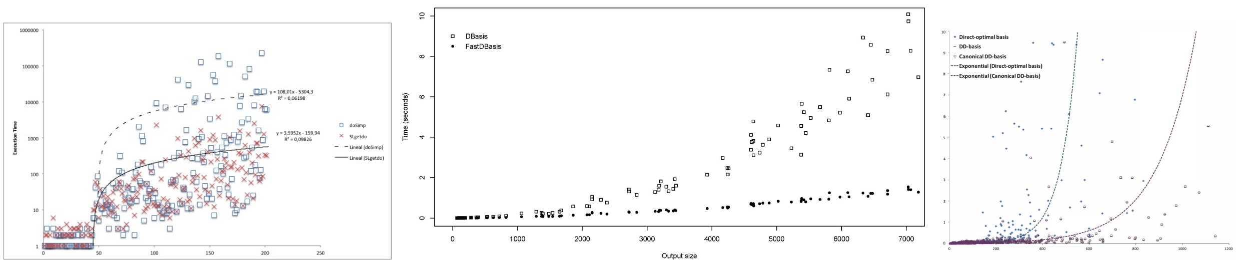

Figure 1: Results of comparisons

References

[ANR13] K. V. Adaricheva, J. B. Nation, and R. Rand. Ordered direct implicational basis of a finite closure system. *Discrete Applied Mathematics*, 161 (6):707–723, 2013.

[1] W W. Armstrong, *Dependency structures of data base relationships*, Proc. IFIP Congress. North Holland, Amsterdam: 580–583, 1974.

[2] K. Bertet, M. Nebut, *Efficient algorithms on the Moore family associated to an implicational system*, DMTCS, 6(2): 315–338, 2004.

[3] K. Bertet, B. Monjardet, *The multiple facets of the canonical direct unit implicational basis*, Theor. Comput. Sci., 411(22-24): 2155–2166, 2010.

[4] B. Ganter, *Two basic algorithms in concept analysis*, Technische Hochschule, Darmstadt, 1984.

[5] A. Mora, M. Enciso, P. Cordero, and I. Fortes: *Closure via functional dependence simplification*, I. J.of Computer Mathematics. **89(4)** (2012) 510–526.

[6] A. Mora, M. Enciso, P. Cordero, and I. Pérez de Guzmán: *An Efficient Preprocessing Transformation for Functional Dependencies Sets Based on the Substitution Paradigm.* LNCS. **3040** (2004) 136–146.

[7] E. Rodríguez-Lorenzo, K. Bertet, P.Cordero, M.Enciso, and A. Mora: *The Direct-optimal basis via reductions.* Concept Lattices and Their Applications. (2014) 145–156.

[8] E. Rodríguez-Lorenzo, K. V. Adaricheva, P. Cordero, M. Enciso, and A. Mora: *From an implicational system to its corresponding d-basis.* Proceedings of the Twelfth International Conference on Concept Lattices and Their Applications. (2015) 217–228.

[9] E. Rodríguez-Lorenzo, K. Bertet, P.Cordero, M.Enciso, A. Mora, and M. Ojeda-Aciego: *From Implicational Systems to Direct-Optimal bases: A Logic-based Approach.* Applied Mathematics & Information Sciences. **2L** (2015) 305–317.

[10] E. Rodríguez-Lorenzo, P. Cordero, M. Enciso, A. Mora: *Canonical dichotomous direct bases.* Information Science. **376** (2017) 39–53.

[11] E. Rodríguez-Lorenzo, K. Adaricheva, P. Cordero, M. Enciso, A. Mora: *Formation of the D-Basis from implicational system using Simplification Logic.* Accepted in International Journal of General Systems. 2017.

Non-standard cuts in poset valued settings and applications

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Two types of standard cuts are important tools in dealing with fuzzy structures. In this presentation, these structures will be mostly poset-valued and in a special case lattice-valued. Usually cuts satisfy the analogous crisp properties as the related fuzzy structures and this approach of investigation is called cutworthy. Lattice (and poset) valued fuzzy sets can be represented by cuts [1,4,5] in various ways. Recently, non-standard cuts have also been investigated and their usage is proposed in different applications. If A is a set (or the underlying set of an algebraic structure), $(P, \leq)$ a poset and $\mu : A \to P$ a poset valued fuzzy set, then for $p \in P$, a cut is a subset μ_p^ρ of A defined by: $\mu_p^\rho = \{x \in A \mid \mu(x)\rho p\}$, where ρ is $\geq$ or one of the relations related to $\leq$ (e.g. $>$, $\not<$, $\not\leq$). Non-standard cuts were investigated in [2] and their main properties were presented. In particular in [3] a complete relationship between convexity of poset valued mappings and convexity of their standard and non-standard cuts is provided. Although the results are up to now mostly theoretical, possible applications in image processing, algorithms for the fuzzy convex hull and in objects in multi-dimensional digital spaces are anticipated. Here we shall present and analyze results connected to non-standard cuts and discus examples of applications.

References

[1] E. Horváth, B. Šešelja, A. Tepavčević: *Cut approach to invariance groups of lattice-valued functions*, Soft Computing **21** (2017) 853–859.

[2] V. Janiš, B. Šešelja, A. Tepavčević: *Non-standard cut classification of fuzzy sets*, Information Sciences **177** (2007) 161–169.

[3] V. Janiš, B. Šešelja, A. Tepavčević: *Poset valued convexities*, submitted.

[4] B. Šešelja, D. Stojić and A. Tepavčević: *On existence of P-valued fuzzy sets with a given collection of cuts*, Fuzzy Sets and Systems **161** (2010) 763–768.

[5] M. Gorjanac Ranitović and A. Tepavčević: *General form of lattice-valued fuzzy sets under the cutworthy approach*, Fuzzy Sets and Systems **158** (2007) 1213–1216.

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Attribute reduction methods based on a discernibility matrix are outperformed by basic clarification and reduction

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We analyze methods of attribute reduction based on discernibility matrix which seem to enjoy increased popularity nowadays, based on number of recent publications on these methods. Interestingly enough, any proper complexity analysis or comparison with known methods has not been done. We fill this gap and show that the these methods are strictly inferior to the basic clarification and reduction.

Let $\langle X, Y, I\rangle$ be a formal context. A set $Z \subseteq Y$ of attributes is called *consistent* if the concept lattice $\mathcal{B}(X, Z, I)$ is isomorphic to the concept lattice $\mathcal{B}(X, Y, I)$. An *attribute reduct* is a minimal consistent set.

The attribute reducts can be found by means of clarification and reduction described in the basic literature on FCA [1]. Specifically, we first *clarify* the context (attributes only); i.e we remove duplicate columns. Consequently, we try to reconstruct each column as an intersection of all columns which cover the reconstructed one. If the reconstruction succeeds, i.e. the intersection matches the reconstructed column, the corresponding attribute is reducible and can be eliminated. The elimination of reducible attribute is called *reduction*. The attributes which survive the reduction form an attribute reduct. Both, the clarification and the reduction, can be done in time $O(|Y|^2 \cdot |X|)$. We call this method a CR-method.

Zhang *et al.* [3] inspired by RST introduced a method of enumerating all the attribute reducts. This method computes an attribute discernibility set for every pair of formal concepts as a symmetric difference of their intents, i.e. $D(\langle A_1, B_1\rangle, \langle A_2, B_2\rangle) = (B_1 \cup B_2) - (B_1 \cap B_2)$. Let $\mathcal{D}$ denote the collection of all attribute discernibility sets in a concept lattice. The attribute discernibility sets $D \in \mathcal{D}$ are used as elementary disjunctions of a CNF formula of so-called discernibility function; i.e. $f(\mathcal{D}) = \bigwedge_{D\in\mathcal{D}} \bigvee_{y\in D} y$. The formula is then transformed into a minimal DNF. Elementary conjunctions of the DNF formula of $f(\mathcal{D})$ correspond to all attribute reducts. This method was improved by Qi [2] who observed that it is enough to consider discernibility sets of direct neighbors in the concept lattice instead of all pairs of formal concepts. We call these methods a DM-methods.

Let us start with time complexity analysis. In Zhang *et al.*'s method we need to compute all formal concepts, the complexity of the first part of the method depends on complexity of enumerating all formal concepts. This can be done in time $O(|X| \cdot |Y|^2 \cdot |\mathcal{B}(X, Y, I)|)$. The consequent computation of all attribute discernibility sets can be done in time $O(|Y| \cdot |\mathcal{B}(X, Y, I)|^2)$. Recall, that the number of formal concepts $|\mathcal{B}(X, Y, I)|$ in the worst case is exponential w.r.t. $n = \min(|X|, |Y|)$. In Qi's method we need to compute entire concept lattice, which can be done in time $O(|X| \cdot |Y|^2 \cdot |\mathcal{B}(X, Y, I)|)$. The number of the computed attribute

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discernibility sets is then the number of edges in the Hasse diagram of $\mathcal{B}(X, Y, I)$; in the worst case, the diagram is n-dimensional hypercube, whose number of edges is $n \cdot 2^{n-1}$. Whence the computation of the attribute discernibility sets is done in time $O(|Y| \cdot \min(|X|, |Y|) \cdot |\mathcal{B}(X, Y, I)|)$. Considering, that the number attributes is usually smaller than the number of objects, the complexity is $O(|Y|^2 \cdot |\mathcal{B}(X, Y, I)|)$. In both cases, we skip the analysis of the CNF to DNF minimal transformation.

Both the DM-methods have exponential time complexity w.r.t. number of objects and attributes. This is not surprising as the number of attribute reducts in formal context can be exponential ($2^{\lceil \frac{|Y|}{2} \rceil}$). While CR-method delivers one attribute reduct, the DM-methods deliver all attribute reducts, thus the this difference in time complexities could seem justifiable. Crucial for our comparison is the following proposition.

Proposition 1. *Let $\{y_1, \ldots, y_k\} \subseteq Y$ be a reduct of $\langle X, Y, I \rangle$ and let $Y_i \subseteq Y$ denote a set of all attributes whose columns are duplicates of y_i's column in $\langle X, Y, I \rangle$ (including y_i) for $i = 1, \ldots, k$. Then*

$$(\bigvee_{y_{1,j} \in Y_1} y_{1,j}) \wedge (\bigvee_{y_{2,j} \in Y_2} y_{2,j}) \wedge \cdots \wedge (\bigvee_{y_{k,j} \in Y_k} y_{k,j})$$

is the minimal CNF formula of discernibility function.

Obviously, the transformation from Proposition 1 can be done easily when A_i's are precomputed from the clarification step of the CR-method. That means that the CR-method can be used to find the CNF formula of discernibility function in polynomial time; and, in addition, it is minimal CNF formula, unlike in the case of the DM-methods. With this result, we can easily make proper comparison. The CR-method computes minimal CNF formula of discernibility function in time $O(|Y|^2 \cdot |X|)$ while the Zhang *et al.*'s DM-method computes it in time $O(|Y| \cdot |\mathcal{B}(X, Y, I)|^2)$ and Qi's DM-method computes it in time $O(|X| \cdot |Y|^2 \cdot |\mathcal{B}(X, Y, I)|)$.

Furthermore, it can be shown that the CR-method can be easily adapted to extensions of FCA where DM-methods were used. We proposed a general approach to such adaptations. We conclude that in any FCA extension where the DM-methods were used, a strictly superior method is available. Although an experimental evaluation is planned as a our future research.

References

[1] Bernard Ganter and Rudolf Wille: *Formal Concept Analysis – Mathematical Foundations.* Springer (1999).

[2] Jian-Jun Qi: *Attribute reduction in formal contexts based on a new discernibility matrix.* Journal of Applied Mathematics and Computing. **30**(1) (2009) 305–314.

[3] Wen-Xiu Zhang, Ling Wei, and Jian-Jun Qi: *Rough Sets, Fuzzy Sets, Data Mining, and Granular Computing. RSFDGrC 2005. Lecture Notes in Computer Science,* **3642** (2005) 157–165.

A Perspective and a Prospective on $\mathcal{K}$-Formal Concept Analysis

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$\mathcal{K}$-Formal Concept Analysis (KFCA) is the flavour of Formal Concept Analysis for contexts with entries in a completed, idempotent semifield. This is a commutative, complete, idempotent semiring with residuated addition and multiplicative inverses. The best known idempotent semifield is, perhaps, the *(completed) schedule algebra*, *morphological algebra* or *(completed) max-plus semifield*, but such semifields always come hand in hand with their order duals, in this case the *(completed) tropical algebra* or *(completed) min-plus semifield.*

Since its inception [1] and further elaboration [2, 3] KFCA has developed into a technique for machine learning evaluation [4], Genomic Data Analysis [5–7], Network Analysis [8], and in general as a theoretical basis for Exploratory Data Analysis [9] where it extends Wille's *Landscapes of Knowledge* [10].

Notice that—along with other extensions [11–14]—it has also expanded the theoretical scope of FCA towards different kinds of Galois Connections [2, 15, 16], spectral considerations [17–19] and in general the deep relation of FCA with linear algebra over semirings [3].

In this talk we also consider several possible directions of research considering extension for positive semifields and other types of analysis for formal contexts, with different applications in mind.

References

[1] F. J. Valverde-Albacete, C. Peláez-Moreno, Towards a generalisation of Formal Concept Analysis for data mining purposes, in: Concept Lattices, no. 3874 in LNAI, Springer, 2006, pp. 161—176.

[2] F. J. Valverde-Albacete, C. Peláez-Moreno, Extending conceptualisation modes for generalised Formal Concept Analysis, Information Sciences 181 (2011) 1888–1909.

[3] F. J. Valverde-Albacete, C. Peláez-Moreno, The Linear Algebra in Formal Concept Analysis over Idempotent Semifields, in: Formal Concept Analysis, no. 9113 in LNAI, Springer, 2015, pp. 97–113.

[4] C. Peláez-Moreno, A. I. García-Moral, F. J. Valverde-Albacete, Analyzing phonetic confusions using Formal Concept Analysis, Journal of the Acoustical Society of America 128 (3) (2010) 1377–1390.

[5] J. González-Calabozo, C. Peláez-Moreno, F. J. Valverde-Albacete, Gene expression array exploration using $\mathcal{K}$-Formal Concept Analysis, in: P. Valtchev, R. Jäschke (Eds.), Proceedings of the ICFCA11, Vol. 6628 of LNAI, Springer, 2011, pp. 119–134.

Acknowledgement CPM & FVA have been partially supported by the Spanish Government-MinECo projects TEC2014-53390-P and TEC2014-61729-EXP in this research.

[6] J. M. Fernández-Calabozo, C. Peláez-Moreno, F. Valverde-Albacete, WebGeneKFCA: an On-line Conceptual Analysis Tool for Genomic Expression Data, in: L. Szathmary, U. Priss (Eds.), Concept Lattices and Applications (CLA 2012), 2012, pp. 345–350.

[7] J. M. González-Calabozo, F. J. Valverde-Albacete, C. Peláez-Moreno, Interactive knowledge discovery and data mining on genomic expression data with numeric formal concept analysis., BMC bioinformatics 17 (1) (2016) 374.

[8] F. J. Valverde-Albacete, C. Peláez-Moreno, Extending the HITS Algorithm over Dioids, in: International Workshop on Social Network Analysis using Formal Concept Analysis, Vol. Accepted, 2015, pp. 1–17.

[9] F. J. Valverde-Albacete, J. M. González-Calabozo, A. Peñas, C. Peláez-Moreno, Supporting scientific knowledge discovery with extended, generalized formal concept analysis, Expert Systems with Applications 44 (2016) 198 – 216.

[10] R. Wille, Conceptual landscapes of knowledge: A pragmatic paradigm for knowledge processing, in: W. Gaul, H. Locarek-Junge (Eds.), Classification in the Information Age, Studies in Classification, Data Analysis, and Knowledge Organization, Springer Berlin Heidelberg, 1999, pp. 344–356.

[11] A. Burusco, R. Fuentes-González, The study of the L-fuzzy Concept Lattice, Mathware and Soft Computing 1 (3) (1994) 209–218.

[12] R. Bělohlávek, Fuzzy Relational Systems. Foundations and Principles, Vol. 20 of IFSR International Series on Systems Science and Engineering, Kluwer Academic, 2002.

[13] I. Düntsch, G. Gediga, Approximation operators in qualitative data analysis, Tech. Rep. CS-03-01, Department of Computer Science, Brock University, St. Catharines, Ontario, Canada (May 2003).

[14] J. Medina, M. Ojeda-Aciego, J. Ruiz-Calviño, Formal concept analysis via multi-adjoint concept lattices, Fuzzy Sets And Systems 160 (2) (2009) 130–144.

[15] F. J. Valverde-Albacete, C. Peláez-Moreno, Further Galois connections between semimodules over idempotent semirings, in: J. Diatta, P. Eklund (Eds.), Proceedings of the 4th Conference on Concept Lattices and Applications (CLA 07), Montpellier, France, 2007, pp. 199–212.

[16] F. J. Valverde-Albacete, C. Peláez-Moreno, The linear algebra in extended formal concept analysis over idempotent semifields, in: K. Bertet, D. Borchmann, P. Cellier, S. Ferré (Eds.), Formal Concept Analysis, Springer, 2017, pp. 1–16.

[17] F. J. Valverde-Albacete, C. Pelaez-Moreno, Spectral lattices of $(R_{max,+})$-formal contexts, in: Concept Lattices. Proceedings of the International Conference on Formal Concept Analysis, ICFCA 08, Vol. LCNS 4933, 2008, pp. 124–139.

[18] F. J. Valverde-Albacete, C. Peláez-Moreno, The spectra of irreducible matrices over completed idempotent semifields, Fuzzy Sets and Systems 271 (2015) 46–69.

[19] F. J. Valverde-Albacete, C. Peláez-Moreno, The spectra of reducible matrices over complete commutative idempotent semifields and their spectral lattices, International Journal of General Systems 45 (2) (2016) 86–115.

Reducts and Bireducts considering Tolerance Relations

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One of the most important goals when we try to extract information from a database, in which we have a set of objects and a set of attributes (properties), is to reduce the number of unnecessary attributes [3–5, 7, 9, 14]. This reduction must prevent the occurrence of incompatibilities and it must eliminate existing noise or repeated information. In Rough Set Theory (RST) [8], bireducts were introduced as a successful solution for this problem, which consists of reducing the set of attributes and the set of objects at the same time [1, 2, 6, 10–13].

We also consider a tolerance relationship to build the reducts and bireducts in RST, providing a more flexible framework for computing such sets. In order to compute the reducts and bireducts we have presented a characterization based on the discernibility function. We can obtain information reducts and information bireducts, as well as decision reducts and decision bireducts. This distinction is given by the kind of database which we are working on. According to RST nomenclature, an information system describe a general knowledge, meanwhile a decision system have an extra distinguish attribute, the decision attribute.

The philosophy of reducts and bireducts in RST will be considered in the Formal Concept Analysis framework in further research.

References

[1] M. Benítez, J. Medina, and D. Ślęzak. Delta-information reducts and bireducts. In J. M. Alonso, H. Bustince, and M. Reformat, editors, *2015 Conference of the International Fuzzy Systems Association and the European Society for Fuzzy Logic and Technology (IFSA-EUSFLAT-15), Gijón, Spain*, pages 1154–1160. Atlantis Press, 2015.

[2] M. Benítez, J. Medina, and D. Ślęzak. Reducing information systems considering similarity relations. In J. Kacprzyk, L. Koczy, and J. Medina, editors, *7th European Symposium on Computational Intelligence and Mathematices (ESCIM 2015)*, pages 257–263, 2015.

[3] M. E. Cornejo, J. Medina, and E. Ramírez-Poussa. Attribute reduction in multi-adjoint concept lattices. *Information Sciences*, 294:41 – 56, 2015.

[4] M. E. Cornejo, J. Medina, and E. Ramírez-Poussa. On the use of irreducible elements for reducing multi-adjoint concept lattices. *Knowledge-Based Systems*, 89:192–202, 2015.

Acknowledgement [1] Partially supported by the State Research Agency (AEI) and the European Regional Development Fund (FEDER) project TIN2016-76653-P.
[2] Partially supported by Polish National Science Centre – grant DEC-2012/05/B/ST6/03215 and by Polish National Centre for Research and Development – grants: PBS2/B9/20/2013 in frame of Applied Research Programmes and O ROB/0010/03/001 in frame of Defence and Security Programmes and Projects.

[5] W. S. Du and B. Q. Hu. Attribute reduction in ordered decision tables via evidence theory. *Information Sciences*, 364-365:91 – 110, 2016.

[6] N. Mac Parthalain and R. Jensen. Simultaneous feature and instance selection using fuzzy-rough bireducts. In *Fuzzy Systems (FUZZ), 2013 IEEE International Conference on*, pages 1–8, July 2013.

[7] J. Medina. Relating attribute reduction in formal, object-oriented and property-oriented concept lattices. *Computers & Mathematics with Applications*, 64(6):1992–2002, 2012.

[8] Z. Pawlak. Rough sets. *International Journal of Computer and Information Science*, 11:341–356, 1982.

[9] M.-W. Shao and K.-W. Li. Attribute reduction in generalized one-sided formal contexts. *Information Sciences*, 378:317 – 327, 2017.

[10] D. Ślęzak and A. Janusz. Ensembles of bireducts: Towards robust classification and simple representation. In T.-h. Kim, H. Adeli, D. Ślęzak, F. Sandnes, X. Song, K.-i. Chung, and K. Arnett, editors, *Future Generation Information Technology*, volume 7105 of *Lecture Notes in Computer Science*, pages 64–77. Springer Berlin Heidelberg, 2011.

[11] D. Ślęzak and A. Janusz. Ensembles of bireducts: Towards robust classification and simple representation. In T.-h. Kim, H. Adeli, D. Ślęzak, F. Sandnes, X. Song, K.-i. Chung, and K. Arnett, editors, *Future Generation Information Technology*, volume 7105 of *Lecture Notes in Computer Science*, pages 64–77. Springer Berlin Heidelberg, 2011.

[12] S. Stawicki and D. Ślęzak. Recent advances in decision bireducts: Complexity, heuristics and streams. *Lecture Notes in Computer Science*, 8171:200–212, 2013.

[13] S. Stawicki and D. Ślęzak. Recent advances in decision bireducts: Complexity, heuristics and streams. In P. Lingras, M. Wolski, C. Cornelis, S. Mitra, and P. Wasilewski, editors, *Rough Sets and Knowledge Technology*, volume 8171 of *Lecture Notes in Computer Science*, pages 200–212. Springer Berlin Heidelberg, 2013.

[14] X. Wang and W. Zhang. Relations of attribute reduction between object and property oriented concept lattices. *Knowledge-Based Systems*, 21(5):398–403, 2008.

The use of directional monotonicity in edge detection

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In recent times, there is a huge interest in the study of generalized forms of monotonicity [3], which allows to define and/or cover many functions which, not being aggregation functions because they are not monotone in an usual sense, are of great interest for applications in fields such as image processing, classification or decision making.

One important step in this direction has been the introduction of the notion of pre-aggregation function [4], which is a function with the same boundary conditions as an usual aggregation function, but for which only monotonicity along some fixed direction is required. This notion has shown itself specially useful for classification problems [4], and it has also allowed to include in a common framework some relevant operators outside from the scope of aggregation functions, as it is the case, in particular, of the mode.

The requirement of monotonicity along a fixed direction which is the same for every considered point in the unit hypercube is, however, still too strict for application in fields such as image processing, for instance. When dealing with edge detection in an image, for instance, relevant directions to be considered may change from one pixel (point) to another one [1].

Taking into account this consideration, among others, in this work we introduce the notion of an ordered directionally monotone function as follows [2]:

Definition 2. *Let $F : [0,1]^n \to [0,1]$ be a fusion function and let $\vec{r} \neq \vec{0}$ be an n-dimensional vector. F is said to be ordered directionally (OD) $\vec{r}$- increasing if for any $\mathbf{x} \in [0,1]^n$, for any $c > 0$ and for any permutation $\sigma : \{1,\ldots,n\} \to \{1,\ldots,n\}$ with $x_{\sigma(1)} \geq \cdots \geq x_{\sigma(n)}$ and such that*

$$1 \geq x_{\sigma(1)} + cr_1 \geq \cdots \geq x_{\sigma(n)} + cr_n \geq 0$$

it holds that

$$F(\mathbf{x} + c\vec{r}_{\sigma^{-1}}) \geq F(\mathbf{x})$$

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where $\vec{r}_{\sigma^{-1}} = (r_{\sigma^{-1}(1)}, \ldots, r_{\sigma^{-1}(n)})$.

Analogously, F is said to be ordered directionally $\vec{r}$- decreasing if for any $\mathbf{x} \in [0,1]^n$, for any $c > 0$ and for any permutation $\sigma : \{1,\ldots,n\} \to \{1,\ldots,n\}$ with $x_{\sigma(1)} \geq \cdots \geq x_{\sigma(n)}$ and such that

$$1 \geq x_{\sigma(1)} + cr_1 \geq \cdots \geq x_{\sigma(n)} + cr_n \geq 0$$

it holds that

$$F(\mathbf{x} + c\vec{r}_{\sigma^{-1}}) \leq F(\mathbf{x})$$

where $\vec{r}_{\sigma^{-1}} = (r_{\sigma^{-1}(1)}, \ldots, r_{\sigma^{-1}(n)})$

In this way, the direction along which monotonicity is considered may vary from one point to another, although it is not different for every point in the unit hypercube. This possibility makes the notion of OD monotone function very useful for problems where directionality plays a key role. In this sense, in this work we discuss the construction of a family of edge detectors making use of OD monotone functions and we show that such detectors have a very good behaviour when compared to other ones that can be found in the literature.

References

[1] H. Bustince, E. Barrenechea, M. Pagola, J. Fernandez *Interval-valued fuzzy sets constructed from matrices:Application to edge detection*, Fuzzy Set and Systems. **160** (2009) 1819–1840.

[2] H. Bustince, E. Barrenechea, J. Lafuente, M. Sesma-Sara, R. mesiar, A. Kolesárová, *Ordered directionally monotone functions. Justification and application*, IEEE Transactions on Fuzzy Systems, submitted.

[3] H. Bustince, A. Kolesárová, J. Fernandez, and R. Mesiar, *Directional monotonicity of fusion functions*, European Journal of Operational Research, **244** (2015) 300–308.

[4] G. Lucca, J. Sanz, G. P. Dimuro, B. Bedregal, R. Mesiar, A. Kolesárová, and H. Bustince, *Pre-aggregation functions: construction and an application*, IEEE Transactions on Fuzzy Systems, **24** (2016) 260–272.

Automatic License Plate Recognition

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In this work, we deal with image processing, namely with the car license plate recognition task which includes three sub-tasks: car plate position estimation, characters segmentation, and characters recognition. We discuss that a solution design of the sub-tasks is mainly dependent on a point of view to an image and we consider three points of views (POW): mathematical (M), information technology (IT) and human (H). The goal of the work is to discuss advantages and disadvantages of different approaches categorized for the three points of views in solving the chosen task.

At first, solution based on F-transform pattern matching algorithm [3] will be presented. The algorithm is representative of IT POW and was designed originally for a general pattern matching including strings, sound, images etc. and consists of two steps, a database pre-processing and a searching of a pattern in the pre-processed database. In our task, we put general car license plates into the database and set an input image as the pattern in order to determine parts inside it which are similar to the database images, i.e., license plates to solve the car plate position estimation sub-task. The same algorithm is used for the character segmentation and the character recognition with the difference that the database consists of particular characters instead of license plates.

The discussion is pointed to comparison of the F-transform based solution with solutions based on particular methods such as Projection Histograms (IT POW) [1], Convolution Neural Networks (H POW) [4], Stroke Width Transform (IT POW) [2], Zhao's Texture Feature (M POW) [7], Tesseract (IT POW) [6] and Local Binary Patterns with SVM (IT and M POW) [5]. The comparison is realized for learning time, processing time, success rate, and generality of the solution with the conclusion that methods which are based on a prior knowledge of the problem and designed for a certain problem do not necessarily guarantee the best results.

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References

[1] Deb, Kaushik, Hyun-Uk Chae, and Kang-Hyun Jo: *Vehicle License Plate Detection Method Based on Sliding Concentric Windows and Histogram.* JCP 4.8 (2009) 771–777.

[2] Epshtein, Boris, Eyal Ofek, and Yonatan Wexler: *Detecting text in natural scenes with stroke width transform.* Computer Vision and Pattern Recognition (CVPR), 2010 IEEE Conference on. IEEE (2010) 2963–2970.

[3] Hurtik, Petr, Petra Hodakova, and Irina Perfilieva: *Approximate Pattern Matching Algorithm.* International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems. Springer International Publishing (2016) 577–587.

[4] Li, Hui, and Chunhua Shen: *Reading Car License Plates Using Deep Convolutional Neural Networks and LSTMs.* arXiv preprint arXiv:1601.05610 (2016).

[5] Liu, Lixia, et al.: *Simplified local binary pattern descriptor for character recognition of vehicle license plate.* Computer Graphics, Imaging and Visualization (CGIV), 2010 Seventh International Conference on. IEEE (2010) 157–191.

[6] Smith, Ray: *An overview of the Tesseract OCR engine.* Document Analysis and Recognition, 2007. ICDAR 2007. Ninth International Conference on. Vol. 2. IEEE (2007) 629–633.

[7] Zhao, Jia-Le , Hai-Gen Min, Xiao-Chi Li and Yong Pan: *A license plate recognition algorithm under low illumination environment.* 2015 IEEE First International Smart Cities Conference (ISC2), Guadalajara (2015) 1–6. doi: 10.1109/ISC2.2015.7366171

Fuzzy Black Top-Hat and Hit-or-Miss transformations and their applications

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Soft computing techniques and, in particular, fuzzy mathematical morphology, has been used to deal with the uncertainty typically present in images, providing state-of-the-art results in many applications. This paradigm is based on the use of fuzzy morphological operators generated by using fuzzy conjunctions and fuzzy implication functions (see [5] for further details). From the four basic operators such as fuzzy dilation, erosion, closing and opening, more complex operators can be defined. Of paramount importance are the fuzzy black Top-Hat and Hit-or-Miss transformations defined as follows:

Definition 3 ([5]). *Let A be a grey-level image, B a structuring element, C a fuzzy conjunction and I a fuzzy implication function. Then, the* Fuzzy Black Top-Hat *of A by B is the grey-level image given by*

$$\rho^d_{C,I}(A,B) = \mathcal{C}_{C,I}(A,B) - A,$$

where $\mathcal{C}_{C,I}(A,B)$ denotes the fuzzy closing.

Definition 4 ([6]). *Let A be a grey-level image, B a structuring element, C a fuzzy conjunction, I a fuzzy implication function and N a strong fuzzy negation. The* fuzzy morphological hit-or-miss transform (FMHMT) *of the grey-level image A by $B = (B_1, B_2)$ is defined, for any $y \in d_A$, by*

$$\mathrm{FMHMT}_{C,I,N}(A,B)(y) = C\left(E_I(A,B_1)(y), E_I(N(A),B_2)(y)\right)$$

where $N(A)(x) = N(A(x))$ for all $x \in d_A$ and $E_I(A,B)$ denotes the fuzzy erosion.

The fuzzy black top-hat transformation extracts the dark structures completely removed by the fuzzy closing, contrasting them from the background. On the other hand, the FMHMT can be used to find patterns with a given shape and size in an image. It provides the degree of similarity of the desired pattern in each location. Such similarity is based on the extent to which the foreground structuring element B_1 is included in the image A, and the background structuring element B_2 is included in $N(A)$.

Both operators, in addition to having a strong theoretical background, have been already used successfully in several applications. In [2], two general-purpose curvilinear object detectors based on the aforementioned operators are proposed. These detectors may serve as building blocks for application-specific systems and can be customised depending on the width of the objects of interest. In the first column of Figure 2-(a), we can see how the FMHMT-based detector is able to detect the wrinkles in a human face.

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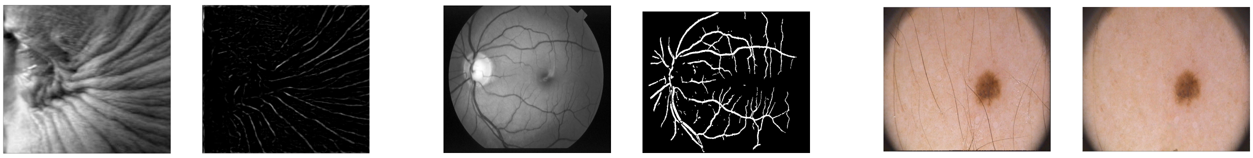
This capacity of detecting curvilinear objects makes the fuzzy Black Top-Hat very appropriate for segmenting vessels in retinal images. This segmentation is the first step towards an automatic diagnosis and in-depth study of retinal images to aid ophthalmologists. Vessels, which are characterised as long, thin regions darker then their background, are enhanced through a fuzzy Black Top-Hat based algorithm that presents good performance measures but still retains a real-time response [1]. In the second column of Figure 2-(b), the result obtained by this real-time detector in the image 235 of the STARE database is shown.

Another application of this operator is the reduction of noise in dermoscopic imagery towards the diagnosis and classification of skin lesions. The presence of bubbles caused by the immersion fluid and skin hair in the image can hinder the diagnosis. Since colors are one of the most important features in these images, a novel paradigm of fuzzy mathematical morphology, called soft color morphology, able to cope with color images, was introduced in [4]. Using the soft color morphology operators and in particular, the soft Black Top-Hat operator, an effective hair removal algorithm for dermoscopic imagery was presented in [3] yielding competitive results. It is able to remove the hair without compromising fine details and texture.



(a) Curvilinear detector (b) Vessels segmentation (c) Skin hair removal

Figure 2: Some results, along with the original image, obtained by fuzzy Black Top-Hat and Hit-or-Miss transforms in some applications.

The above applications ensure the importance of both the fuzzy Black Top-Hat and the Hit-or-Miss transforms. Further theoretical analysis and experimental results are being carried out to develop new algorithms based on these transforms for other image processing applications.

References

[1] P. Bibiloni, M. González-Hidalgo, S. Massanet: *A Real-time Fuzzy Morphological Algorithm for Retinal Vessel Segmentation.* Submitted to Information Sciences.

[2] P. Bibiloni, M. González-Hidalgo, S. Massanet: *General-Purpose Curvilinear Object Detection with Fuzzy Mathematical Morphology.* Submitted to Applied Soft Computing.

[3] P. Bibiloni, M. González-Hidalgo, S. Massanet: *Skin Hair Removal in Dermoscopic Images using Soft Color Morphology.* Accepted in AIME 2017.

[4] P. Bibiloni, M. González-Hidalgo, S. Massanet: *Soft Color Morphology.* Accepted in FUZZ-IEEE 2017.

[5] M. Nachtegael, E. E. Kerre: *Classical and fuzzy approaches towards mathematical morphology,* in: E. E. Kerre and M. Nachtegael, editors, Fuzzy techniques in image processing, **52** in Studies in Fuzziness and Soft Computing, chapter 1, pp. 357. Physica-Verlag, 2000.

[6] M. González-Hidalgo, S. Massanet, A. Mir, D. Ruiz-Aguilera: *A fuzzy morphological hit-or-miss transform for grey-level images. A new approach.* Fuzzy Sets and Systems. **286** (2016) 30-65.

A General Use of the Pattern Matching Algorithm

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The goal is to present a general algorithm suitable for exact and approximate pattern matching universally applicable for arbitrary dimensional data. Our aim was to develop such a matching algorithm that is as much effective with respect to the success rate and computational speed as possible. In order to process large data and achieve the sufficient computational speed, we work with reduced approximated data instead of the original data. Therefore, our algorithm is based on the technique of Fuzzy transform (F-transform) [1] that is one of aggregation methods which reduces the volume of data. Originally, the F-transform based pattern matching algorithm was developed for string matching [2], later extended for image matching [3] and for r-dimensional object matching in general [4].

Till now, the applicability of the algorithm was mainly studied for one and two-dimensional data - strings and images. We tested the algorithm for such data on large databases and compared the results with other commonly used and available methods. One of the goals was an investigation of all the methods with respect to their general use, i.e., different dimensions, exact / approximate matching or both, pre-processing needed, GPU version possibilities, etc. The other goal was their testing with respect to the success rate and computational time for matching of particular patterns in the databases. The final conclusion was that our algorithm based on the F-transform was the most general and fastest one in comparison to the others tested, see [5].

The practical use of the algorithm was demonstrated on the task of a real-time object tracking by a drone [6]. The drone streams a video into a mobile phone where it is split into images. A user denotes an object in the image which is considered as a pattern and should be tracked by the drone. The pattern is then processed by the F-transform based pattern matching algorithm and its position is taken and used for the drone control in order to track the real object in the scene. This application shows the robustness of the pattern matching algorithm to noise or object overlapping and takes advantage of its computational speed that are important aspects of the real application processing.

References

[1] I. Perfilieva: *Fuzzy transforms: Theory and applications.* Fuzzy Sets and Systems. **157** (2006) 993–1023.

[2] P. Hurtik, P. Hodáková, I. Perfilieva: *Fast string searching mechanism.* In: Proceedings of the 2015 Conference of the International Fuzzy Systems Association and the European Society for Fuzzy Logic and Technology. AISR, **89** (2015) 412–418.

Acknowledgement This work has been supported by the project "LQ1602 IT4Innovations excellence in science".

[3] P. Hurtik, P. Hodáková: *FTIP: A tool for an image plagiarism detection.* In: 7th International Conference of Soft Computing and Pattern Recognition (SoCPaR). (2015) 42–47.

[4] P. Hurtik, P. Hodáková, I. Perfilieva: *Approximate pattern matching algorithm.* In: International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems. IPMU 2016. Communications in Computer and Information Science. Springer, **610** (2016) 577–587.

[5] P. Hurtik, P. Števuliáková: *Pattern matching: overview, benchmark and comparison with the F-transform general matching algorithm.* Soft Computing. (*submitted*)

[6] M. Vajgl, P. Hurtik, P. Števuliáková: *Drone real-time control based on pattern matching.* IFSA-SCIS 2017. (*accepted*)

Multistage stochastic dominance risk averse measures in mathematical optimization under uncertainty, and industrial experiences

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An optimization model under uncertainty in he main parameters deals with finding feasible solutions to minimize the objective function (say, cost) expected value in the scenarios alone in a given time horizon is so-named risk neutral (RN) strategy. It can provide very good results, mainly, when it is compared with the expected value strategy. However, the variability of the cost is not fully considered and, then, the RN solution may induce a high negative impact in low-probability high-cost scenarios. Some approaches for cost risk reduction measures have appeared in the literature since more than 25 years ago. In this talk, the time inconsistent and time consistent versions of the Stochastic Dominance (SD) risk averse measure are presented and computationally tested on real-life large-sized instances from different sectors. A SD risk averse measure controls the excess of the value of a function under consideration (one of then, usually, is the cost function, although others could be also the subject is an goal programming manner) over a set of modeler-driven increasingly cost thresholds as targets in the scenarios. That control basically consists of a mixture of the first-order SD (also called chance constrained) functional and the second-order SD functional related to a set of modeler-driven profiles in the nodes of a modeler-driven subset of periods in the time horizon for each function in a given set. Each profile is included by a cost threshold, a bound on the expected cost excess over the threshold in the scenario group with a one-to-one correspondence with the nodes in the selected periods, and a bound on the failure probability of any of those scenarios to satisfying the cost threshold.

References

[1] L.F. Escudero, A. Garín, M. Merino and G. Pérez. *On time stochastic dominance induced by mixed-integer linear recourse in multistage stochastic programs.* European Journal of Operational Research. **249** (2016) 164–176.

[2] L.F. Escudero, J.F. Monge and D. Romero-Morales. *An SDP approach for multi-period mixed 0-1 linear programming models with stochastic dominance constraints for risk management.* Computers & Operations Research. **58** (2015) 32–40.

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Stochastic Predicted Bands: A Novel Interval-Based Method for Decision-Making

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In this work, an interval-based method is presented to consider the uncertainty of decision-making variables without knowledge of the probability density function. This method is called Stochastic Predicted Bands (SPB) [1]- [3]. In SPB, modeling of uncertainty is done by bands based on the prediction of stochastic variables. Besides, an auxiliary parameter is defined to provide flexibility to the decision-maker to be optimistic or conservative. Hence, applying the *optimistic coefficient* to the SPB method results in the enhancement of its performance. This new method is called Modified Stochastic Predicted Bands (MSPB).

Stochastic Predicted Bands Method

Step 1

This model consists of two stages. Uncertainty of the variables is not considered in the first stage. However, the uncertainty of variables is considered in the second stage. The first stage variables play an important role in converging the second stage variables to their optimum decisions when their uncertainties converge to zero. Hence, the first stage variables should be determined in the first step.

Step 2

In this method, the uncertainty of variables is considered based on their predicted amounts. Hence, short-term forecasting of variables is done in the second step. Besides, σ_{up} and σ_{down} are parameters that are defined to state the amounts of upper and lower variances of the predicted variable in comparison with its actual amount, respectively.

Step 3

In this step, the difference between the first stage amount of variables, E_t^1, and their predicted amount in each time, E_t^{pred} is determined.

$$D_t = E_t^1 - E_t^{pred} \tag{1}$$

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Step 4

According to the state of D_t, the second stage decision-making variables, E_t^2, are limited to the maximum and minimum bands. If D_t is positive, it means that the amount of first stage variable is more than the predicted amount. Hence, the amount of second stage variable should be greater than the predicted amount to converge to the amount of first stage variable. If D_t is negative, the predicted amount of the variable is more than the first stage one. Hence, as the second stage variable likes to converge to the amount of its first stage one, the second stage variable will be limited to the predicted amount as its maximum band. Therefore, the minimum limitation of the second stage variable will be based on the down variance of its prediction because the variables predicted amount is more than its first stage amount. Equation (2) is defined to clarify the above explanations:

$$\begin{cases} E_t^{pred} \leq E_t^{rt} \leq E_t^{pred} + \sigma_{up} & D_t \geq 0 \\ E_t^{pred} - \sigma_{down} \leq E_t^{rt} \leq E_t^{pred} & D_t \leq 0 \end{cases} \tag{2}$$

Modified Stochastic Predicted Bands Method

One of the drawbacks of the SPB method is that the uncertainty of the stochastic variables cannot be modeled completely based on the predicted bands. Also, the second stage variables tend to converge to the maximum and minimum bands based on their amounts in the first stage. In this section, an auxiliary parameter is defined as a slack parameter in order to give freedom to the decision-maker to apply its knowledge regarding the stochastic behavior of the uncertain variable. This parameter is called an *optimistic coefficient*, α, and its amount can be between 0 and 1. This method is called MSPB. Equation (3) is the modified version of equation (2), if the MSPB method is utilized in the problem.

$$\begin{cases} E_t^{pred}\alpha + (E_t^{pred} - \sigma_{down})(1-\alpha) \leq E_t^{rt} & \\ \leq (E_t^{pred} + \sigma_{up})\alpha + E_t^{pred}(1-\alpha) & D_t \geq 0 \\ (E_t^{pred} - \sigma_{down})\alpha + E_t^{pred}(1-\alpha) \leq E_t^{rt} & \\ \leq E_t^{pred}\alpha + (E_t^{pred} + \sigma_{up})(1-\alpha) & D_t \leq 0 \end{cases} \tag{3}$$

References

[1] Amin Shokri Gazafroudi, F. Prieto-Castrillo, and J. M. Corchado: *Residential Energy Management Using a Novel Interval Optimization Method. Accepted:* 4^{th} Int. Conf. on Con., Dec. and Inf. Tech. (2017).

[2] Amin Shokri Gazafroudi, T. Pinto, F. Prieto-Castrillo, J. Prieto, J. M. Corchado, A. Jozi, Z. Vale, G. K. Venayagamoorthy: *Organization-based Multi-Agent Structure of the Smart Home Electricity System. Accepted:* IEEE Cong. on Evolut. Comp. (2017).

[3] Amin Shokri Gazafroudi, F. Prieto-Castrillo, T. Pinto, A. Jozi, Z. Vale: *Economic Evaluation of Predictive Dispatch Model in MAS-based Smart Home. Accepted:* 15^{th} Int. Conf. on Prac. App. of Agents and Multi-Agent Sys. (2017).

Regression with fuzzy/linguistic rules

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There are two main branches of machine learning and data mining. The first one consists of various approaches for building a model of data. The examples are decision trees, support vector machines, linear regression etc. The aim of this methods is to build a model for regression or classification. The second branch is called explorative data mining and consists mainly of clustering and association rule mining. Explorative data mining usually does not aim at building any model, though subsequent steps in the analysis might lead to some models, but rather tries to explore and find some interesting structural information about data.

On the border between explorative and model building data science is associational classification (i.e. classification with association rules), see [1, Chapter 6.13] and references therein. Similarly to associational classification, it is possible to develop associational regression, which is using fuzzy association rules as linguistic IF-THEN rules [2]. Theses rules combined with an input derive a fuzzy set as a conclusion in some inference system. The output fuzzy set is defuzzified and thus regression is obtained.

In our presentation, we will describe the approach of regression with fuzzy associational rules published in [2]. We will present various strategies for reducing the vast amount of associational rules into processable knowledge base [3, 4].

References

[1] Jiawei Han. *Data Mining: Concepts and Techniques*. Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, 2005.

[2] Jiří Kupka and Pavel Rusnok. Regression analysis based on linguistic associations and perception-based logical deduction. *Expert Systems with Applications*, 67:107–114, 2017.

[3] Michal Burda and Martin Štěpnička. Reduction of fuzzy rule bases driven by the coverage of training data. In *Proc. 16th World Congress of the International Fuzzy Systems Association and 9th Conference of the European Society for Fuzzy Logic and Technology, IFSA-EUSFLAT*, 2015.

[4] Pavel Rusnok. Probabilistic coverage of linguistic if-then rules. In *IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, pages 798–803. IEEE, 2016.

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Short-term Forecasting of Price-responsive Loads Using Inverse Optimization

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A method for forecasting the aggregate demand of a cluster of price-responsive consumers of electricity is discussed in this presentation [1]. The price-response of the aggregation is modeled by an optimization problem whose defining parameters represent a series of marginal utility curves, and minimum and maximum consumption limits. These parameters are, in turn, estimated from observational data using an approach inspired from duality theory. The resulting estimation problem is nonconvex, which makes it very hard to solve. In order to obtain good parameter estimates in a reasonable amount of time, we divide the estimation problem into a feasibility problem and an optimality problem. Furthermore, the feasibility problem includes a penalty term that is statistically adjusted by cross validation. The proposed methodology is data-driven and leverages information from regressors, such as time and weather variables, to account for changes in the parameter estimates [2]. The estimated price-response model is used to forecast the power load of a group of heating, ventilation and air conditioning systems, with positive results.

References

[1] Author(s): J. Saez-Gallego and J. M. Morales. *Short-term Forecasting of Price-responsive Loads Using Inverse Optimization.* IEEE Transactions on Smart Grid. **In press** (2017).

[2] Author(s): J. Saez-Gallego, J. M. Morales, M. Zugno, and H. Henrik. *A data-driven bidding model for a cluster of price-responsive consumers of electricity.* IEEE Transactions on Power Systems. **No** 31 (2016) 5001–5011.

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Capacity Expansion of Stochastic Power Generation under Two-Stage Electricity Markets

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Energy imbalances due to power forecast errors have a significant impact on both the cost of operating the power system and the profitability of stochastic power generating units. In this paper, we propose a modeling framework to analyze the effect of the costs of these imbalances on the expansion of stochastic power generating units. This framework includes the explicit representation of a day-ahead and a balancing market-clearing mechanisms to properly capture the impact of forecast errors of power production on the short-term operation of a power system. The proposed generation expansion problems are first formulated from the standpoint of a social planner to characterize a perfectly competitive market. We investigate the effect of two paradigmatic market designs on generation expansion planning: a day-ahead market that is cleared following a conventional cost merit-order principle, and an ideal market-clearing procedure that determines day-ahead dispatch decisions accounting for their impact on balancing operation costs. Furthermore, we reformulate the proposed models to determine the optimal expansion decisions that maximize the profit of a collusion of stochastic power producers in order to explore the effects of competition at the investment level. The proposed models are first formulated as multi-level programming problems and then recast, under certain assumptions, as single-level mixed-integer linear or non-linear optimization problems using duality theory. The variability of the forecast of the stochastic power production and the demand level throughout the planning horizon is modeled using yearly duration curves. Likewise, the uncertainty pertaining to power forecast errors is characterized through scenario sets. The main features and results of the proposed models are discussed using an illustrative example and a more realistic case study based on the Danish power system.

Energy and Reserve Scheduling under Uncertain Nodal Net Power Injections: A Two-Stage Adaptive Robust Optimization Approach

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Over the last years, power systems have experienced a steady increase in the use of renewable-based power generation technologies such as wind and photovoltaic generators. In order to cope with the uncertainty and intermittency associated with those environmentally friendly energy sources, increased levels of reserves are required. As a consequence, the scheduling, procurement, and eventual deployment of adequate levels of reserves have become issues of major concern in power system operation.

For scheduling purposes, the system operator solves a unit-commitment-based problem wherein technical and economic aspects of generating units as well as network limitations are accounted for. However, the effect of the transmission network on the deployment of reserves is typically disregarded. Therefore, network limits may lead to the undeliverability of reserves when such ancillary services are called by the system operator to withstand variations in nodal net power injections. Hence, the required corrective actions implemented by the system operator, such as generation and reserve redispatch, may give rise to undesirable cost increases and price spikes.

This presentation describes a new approach relying on adjustable robust optimization to guarantee reserve deliverability in network-constrained generation scheduling under uncertainty. The key idea is to schedule energy and reserves while accounting for the worst-case nodal net power injection under a given user-defined uncertainty set. The robust counterpart is formulated as a mixed-integer trilevel program that is effectively solved by means of a column-and-constraint generation algorithm.

The performance of the proposed approach is examined with the IEEE Reliability Test System. Numerical results are compared with those provided by a conventional model based on forecasted injections and system reserve requirements. Simulations reveal that reserve deployment is successfully guaranteed by the proposed adjustable robust optimization model.

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Toward Cost-Efficient and Reliable Unit Commitment Under Uncertainty

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Large-scale integration of wind farms causes volatile bus net injections. Although these fluctuations are anticipated, their timing, magnitude and duration cannot be predicted accurately. In order to maintain the operational reliability of the system, this uncertainty must be adequately addressed at the day-ahead generation scheduling stage. The ad-hoc reserve rules incorporated in deterministic unit commitment formulations do not adequately account for this uncertainty [1]. Scenario-based stochastic unit commitment formulations model this uncertainty more precisely [2], but require computationally demanding simulations [3]. Interval [4] and robust [5] optimization techniques require less computing resources, but produce overly conservative and thus expensive generation schedules.

The interval unit commitment formulation is computationally more efficient than the stochastic unit commitment because the generation uncertainty of each wind farm is represented by only three non-probabilistic scenarios: the central forecast and the lower bound and the upper bounds. On the other hand, interval unit commitment solutions are more conservative because of the constraints that it imposes on the feasibility of transitions from lower to upper bound, and vice versa, between any two consecutive time periods. Such extreme transitions have a very low probability and can be replaced by less severe ramp constraints. Since scenarios are designed to accurately capture the characteristics of the expected wind output, we argue that the required rampable capacity should be no more than the maximum up and down ramps observed over all stochastic scenarios.

We propose a new improved interval unit commitment formulation [6] that aims at improving the day-ahead reliability unit commitment procedures and combines aspects of stochastic and interval unit commitment. This model takes advantage of the cost-efficient stochastic model and the computational simplicity of the interval model. Effectiveness of the improved interval formulation is demonstrated on extensive case study with various wind penetration levels, wind profiles and controllable generator characteristics. A systematic and rigorous comparative assessment is conducted of the cost and reliability performance of the improved interval, interval, robust and stochastic unit commitment formulations. All the data used for these test cases, as well as the GAMS codes based on [7] for all four formulations are available online at [8].

References

[1] Y. Dvorkin, M. A. Ortega-Vazquez, and D. S. Kirschen: *Assessing flexibility requirements in power systems.* IET Gener., Transm., Distrib. **11** (2014) 1820–1830.

[2] Y. Dvorkin, Y. Wang, H. Pandžić, and D. S. Kirschen: *Comparison of scenario reduction techniques for the stochastic unit commitment.* Proc. IEEE PES General Meeting 2014. 1–5.

[3] S. Takriti, J. R. Birge, and E. Long: *A stochastic model for the unit commitment problem.* IEEE Trans. Power Syst. **3** (1996) 1497–1508.

[4] Y. Wang, Q. Xia, and C. Kang: *Unit commitment with volatile node injections by using interval optimization.* IEEE Trans. Power Syst. **3** (2011) 1705–1713.

[5] D. Bertsimas, E. Litvinov, X. A. Sun, Z. Jinye, and Z. Tongxin: *Adaptive robust optimization for the security constrained unit commitment problem.* IEEE Trans. Power Syst. **1** (2013) 52–63.

[6] H. Pandžić, Y. Dvorkin, T. Qiu, Y. Wang, and D. S. Kirschen: *Toward Cost-Efficient and Reliable Unit Commitment Under Uncertainty.* IEEE Trans. Power Syst. **2** (2016) 970–982.

[7] H. Pandžić, T. Qiu, and D. Kirschen: *Comparison of state-of-the-art transmission constrained unit commitment formulations.* Proc. IEEE PES General Meeting 2013. 1–5.

[8] H. Pandžić, Y. Dvorkin, T. Qiu, Y. Wang, and D. S. Kirschen: *Unit Commitment under Uncertainty - GAMS Models.* Library of the Renewable Energy Analysis Lab (REAL), University of Washington, Seattle, USA. Available at: http://www.ee.washington.edu/research/real/gams_code.html

An Efficient Robust Solution to the Two-Stage Stochastic Unit Commitment Problem

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The large-scale integration of renewable energy sources in many power systems worldwide has made the supply of electricity variable and partly unpredictable. This fundamental change in the nature of the generation-side of power systems has, in turn, spurred the scientific and industrial community into revising current procedures for power system operations. This is the case, for example, of the tool used by system operators to decide the commitment of power plants, that is, to solve the so-called unit commitment (UC) problem.

Within this context, our work [1] proposes a partition-based reformulation of the two-stage unit commitment problem under uncertainty that allows finding unit-commitment plans with different degrees of conservatism as for the eventual power system operating cost. The idea is to approximate the probability distribution by clustering the scenarios in different partitions. Therefore, several clusterization techniques are applied to optimize the partitioning process.

We name this strategy *hybrid* formulation of the UC problem. We assume it is a *hybrid* because it oscillates between the stochastic formulation of the UC since we consider a finite set of scenarios that follow a perfectly known probability distribution and a robust formulation of the UC problem as we minimize the total function cost for its worst-case realization in every partition. Different *hybrid* formulations of the UC problem have been previously considered (e.g. [2] and [3]) in the sense that some of the uncertain parameters are assumed to follow certain probability distributions, while others are solely known to belong to some uncertainty sets. However, what makes our work different is the fact that we assume that the probability distributions of the uncertain parameters (i.e. the wind power production) are known, but that, as it normally occurs in practice, computational tractability only allows us to solve the stochastic unit commitment problem for a scenario-based approximation of such distributions.

Furthermore, we provide two parallelization and decomposition schemes based both on a column-and-constraint generation procedure [4] that drastically reduces the computational time required to solve the proposed UC model. Finally, we test both the quality of the solution provided by our UC model and the performance of the associated solution algorithm on the IEEE 14-node power system and the IEEE 3-Area RTS-96 system. The effectiveness of our proposed method is later compared to a widely applied scenario reduction technique [5], noting an improvement in terms of expected cost and worst-case cost.

References

[1] I. Blanco and J. M. Morales, “An efficient robust solution to the two-stage stochastic unit commitment problem,” *IEEE Transactions on Power Systems*, vol. PP, no. 99, pp. 1–1, 2017.

[2] C. Zhao and Y. Guan, “Unified stochastic and robust unit commitment,” *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 3353–3361, 2013.

[3] C. Zhao and Y. Guan, “Data-driven stochastic unit commitment for integrating wind generation,” *IEEE Transactions on Power Systems*, vol. 31, no. 4, pp. 2587–2596, 2016.

[4] B. Zeng and L. Zhao, “Solving two-stage robust optimization problems using a column-and-constraint generation method,” *Operations Research Letters*, vol. 41, no. 5, pp. 457–461, 2013.

[5] H. Heitsch and W. Römisch, “Scenario reduction algorithms in stochastic programming,” *Computational Optimization and Applications*, vol. 24, pp. 187-206, 2003.

Some observations on paraconsistent degree-preserving fuzzy logics

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In the recent past, formal systems of fuzzy logic, under the umbrella of mathematical fuzzy logic (MFL) [2], have been proposed and studied as suitable tools for reasoning with propositions containing vague predicates. One of their main features is that they allow to interpret formulas in linearly ordered scales of truth values, which makes them specially suited for representing the gradual aspects of vagueness. Particular deductive systems in MFL have been usually studied under the paradigm of (full) *truth-preservation* which, generalizing the classical notion of consequence, postulates that a formula follows from a set of premises if every algebraic evaluation that interprets the premises as true also interprets the conclusion as true. An alternative approach that has recently received some attention is based on the *degree-preservation* paradigm [1, 6], in which a conclusion follows from a set of premises if, for all evaluations, the truth degree of the conclusion is not lower than that of the premises. It has been argued that this approach is more coherent with the commitment of many-valued logics to truth-degree semantics because all values play an equally important rôle in the corresponding notion of consequence [5].

Moreover, while the truth-preserving fuzzy logics are explosive, i.e. from any theory containing a formula φ and its negation $\neg\varphi$ everything follows, in two recent papers [3, 4] some (extensions of) degree-preserving fuzzy logics have been shown to exhibit some well behaved paraconsistency properties. In particular, this is the case of the well-known Łukasiewicz logic Ł, whose degree preserving companion $Ł^{\leq}$ is not explosive, i.e. it is paraconsistent. Actually, the degree-preserving companions of finitely-valued Łukasiewicz logics $Ł_n$ belong to the family of the so-called *logics of formal inconsistency* (**LFI**s).

In this paper we introduce $RPL^{\leq}$, the degree-preserving companion of well-known Rational Pavelka logic RPL (the expansion of Łukasiewicz logic with rational truth-constants as defined by Hájek in [7]) and study some of its properties. The use of truth-constants allows us to explicitly show what is the inferential loss in some reasoning patterns when moving from RPL to its weaker, paraconsistent companion $RPL^{\leq}$.

Recall that the logical consequence relation for $Ł^{\leq}$ is defined as follows [6]: for every set of formulas $\Gamma \cup \{\varphi\}$, $\Gamma \models_{Ł^{\leq}} \varphi$ iff for every evaluation e over the standard MV-algebra $[0, 1]_{MV}$ and every $a \in [0, 1]$, if $a \leq v(\gamma)$ for every $\gamma \in \Gamma$, then $a \leq v(\varphi)$. For this reason $Ł^{\leq}$ is known as the Łukasiewicz logic *preserving degrees of truth*, or the *degree-preserving companion* of Ł. In fact, Ł and $Ł^{\leq}$ have the same tautologies, and for every finite set of formulas $\Gamma \cup \{\varphi\}$ we have:

$$\Gamma \models_{Ł^{\leq}} \varphi \text{ iff } \models_{Ł} \Gamma^{\wedge} \to \varphi,$$

where $\Gamma^{\wedge}$ means $\gamma_1 \wedge \ldots \wedge \gamma_k$ for $\Gamma = \{\gamma_1, \ldots, \gamma_k\}$ (when Γ is empty then $\Gamma^{\wedge}$ is $\top$). It is worth noticing that the usual rule of modus ponens is not sound for $Ł^{\leq}$. However, the logic $Ł^{\leq}$

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admits a Hilbert-style axiomatization with a weaker form of modus ponens. Indeed, by letting the axioms of Ł$^{\leq}$ be the same axioms as Ł and having the following deduction rules:

$$\textbf{(Adj-}\wedge\textbf{)} \quad \frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad\qquad \textbf{(MP-}r\textbf{)} \quad \frac{\varphi \quad \vdash_{Ł} \varphi \to \psi}{\psi}$$

one gets a sound and complete axiomatisation of $\models_{Ł^{\leq}}$ for deductions from a finite set of formulas [1].

Now we can introduce the logic RPL$^{\leq}$. First we extend the language of Ł$^{\leq}$ by introducing a rational truth-constant $\overline{r}$ for every rational $r \in [0,1]$. The notion of logical consequence, $\models^{\leq}_{RPL}$, is defined as $\models_{Ł^{\leq}}$ with the proviso that every evaluation e over the standard MV-algebra $[0,1]_{\mathrm{MV}}$ additionally satisfies $e(\overline{r}) = r$ for every rational $r \in [0,1]$. On the other hand, one gets a sound and finite strong complete axiomatisation for $\models^{\leq}_{RPL}$ just adding to the axiomatic system for Ł$^{\leq}$ the usual booking axioms for truth-constants. Moreover, an analogous Pavelka-style completeness result for RPL$^{\leq}$ can also be obtained: for any set of RPL formulas $T \cup \{\varphi\}$, define:

- *truth degree* of φ in T: $\|\varphi\|^{\leq}_{T} = \inf\{e(T) \to e(\varphi) : e$ RPL-evaluation$\}$,
- *provability degree* of φ from T: $\mid \varphi \mid^{\leq}_{T} = \sup\{r \mid T \vdash^{\leq}_{RPL} \overline{r} \to \varphi\}$,

where $e(T) = \inf\{e(\psi) : \psi \in T\}$.

Theorem 5. *For any set of RPL formulas $T \cup \{\varphi\}$, we have:*

$$\mid \varphi \mid^{\leq}_{T} = \|\varphi\|^{\leq}_{T}.$$

This shows that RPL$^{\leq}$ is well-behaved in a sense. However, we have mentioned above that the usual rule of modus ponens is not sound in Ł$^{\leq}$, and hence neither in RPL$^{\leq}$. Actually, in RPL$^{\leq}$, one can show that the following deduction holds: $\{\varphi, \varphi \to \psi\} \models^{\leq}_{RPL} \overline{0.5} \to \psi$. That is, in RPL$^{\leq}$ we are forced to lower the truth-degree of the conclusion in order to have a sound but weaker modus ponens rule. We will discuss this and other facts about RPL$^{\leq}$ that may be seen as a somewhat questionable price to pay for enjoying a paraconsistent behaviour.

References

[1] F. Bou, F. Esteva, J.M. Font, A. Gil, L. Godo, A. Torrens, and V. Verdú. Logics preserving degrees of truth from varieties of residuated lattices. *Journal of Logic and Computation*, 19(6):1031–1069, 2009.

[2] P. Cintula, P. Hájek, and C. Noguera, editors. *Handbook of Mathematical Fuzzy Logic (in 2 volumes)*, volume 37, 38 of *Studies in Logic, Mathematical Logic and Foundations*. College Publications, London, 2011.

[3] M. E. Coniglio, F. Esteva, and L. Godo. Logics of formal inconsistency arising from systems of fuzzy logic. *Logic Journal of the IGPL* 22(6):880–904, 2014.

[4] R. Ertola, F. Esteva, T. Flaminio, L. Godo, and C. Noguera. Exploring paraconsistency in degree-preserving fuzzy logics. *Soft Computing*,19(3): 531-546 (2015)

[5] J. M. Font. Taking degrees of truth seriously. *Studia Logica*, 91(3):383–406, 2009.

[6] J. M. Font, A. Gil, A. Torrens, and V. Verdú. On the infinite-valued Łukasiewicz logic that preserves degrees of truth. *Archive for Mathematical Logic*, 45(7):839–868, 2006.

[7] P. Hájek. *Metamathematics of fuzzy logic*. Volume 4 of Trends in Logic. Kluwer, Dordrecht, 1998.

On the interpretation of undefined degrees in partial fuzzy logic

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Partial fuzzy logic aims at accommodating fuzzy propositions whose truth degree may not be defined. In [1], a simple partial extension L^* of any propositional ($\triangle$-core) fuzzy logic L has been proposed. In L^*, undefined propositions are assigned an extra truth value $*$, incomparable to ordinary truth degrees from $[0, 1]$ or another L-algebra. The propositional connectives of the underlying fuzzy logic L are extended to a broad array of definable connectives of L^*, which differ in how they act on the extra truth value $*$. The prominent families of L^*-connectives include Bochvar (where $*$ is the absorbing element), Sobociński (where $*$ is the neutral element), Kleene (which keep the original absorbing and neutral elements), and several others.

Since there are several possible meanings of 'undefined' (including: unknown, uncertain, undetermined, as-yet-uncomputed, nonsensical, etc., each governed by different laws), it is important to clarify which of these meanings can be represented by $*$ in partial fuzzy logic, as misapplying the apparatus in inappropriate contexts would yield wrong results. To this end, I will compare partial fuzzy logic to several fuzzy formalisms dealing with similar phenomena, esp. interval-valued and bilattice-valued fuzzy logics, three-valued partial logic, and epistemic modal logic [2–7].

I will show that although the truth values of L^* can be embedded in the sets of truth values of these logics in several ways (e.g., identifying $*$ with either $[0, 1]$ or $\emptyset$ in interval-valued logic), the connectives and laws of all these logics differ from those of L^*. This corroborates the fact that the types of undefinedness captured by these logics are different from the undefinedness formalized in partial fuzzy logic L^*. Consequently, the dummy value $*$ of L^* cannot represent an unknown, uncertain, uncomputed, or underdetermined degree. Rather, it should be interpreted as an 'error code' for a nonsensical or objectively undefined value, analogous to the 'not-a-number' value for division by zero in some programming languages. The variant connectives of L^* then specify how the error is handled as it propagates from subformulae (e.g., as a fatal error by the Bochvar connectives, an irrelevant nonsense by the Sobociński connectives, etc.).

References

[1] Běhounek L., Novák V.: *Towards fuzzy partial logic.* Proceedings of the IEEE 45th ISMVL, Waterloo, Ontario, 2015, pp. 139–144.

[2] Blamey S.: *Partial logic.* In D.M. Gabbay, F. Guenthner: Handbook of Philosophical Logic (2nd Edition), Vol. 16, pp. 261–353. Springer, 2011.

[3] Cornelis C., Arieli O., Deschrijver G., Kerre E.E.: *Uncertainty modeling by bilattice-based squares and triangles.* IEEE Transactions on Fuzzy Systems **15** (2007) 161–175.

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[4] Dubois D.: *On ignorance and contradiction considered as truth-values.* Logic Journal of the IGPL, **16** (2008) 195–216.

[5] Esteva F., Garcia-Calvés P., Godo L.: *Enriched interval bilattices and partial many-valued logics: An approach to deal with graded truth and imprecision.* International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems **2** (1994) 37–54.

[6] Genito D., Gerla G.: *Connecting bilattice theory with multivalued logic.* Logic and Logical Philosophy **23** (2014) 15–45.

[7] Turunen E., Öztürk M., Tsoukiás A.: *Paraconsistent semantics for Pavelka style fuzzy sentential logic.* Fuzzy Sets and Systems **161** (2010) 1926–1940.

Fuzzy set theory
with undefined membership values

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We will present a theory of fuzzy sets that can have undefined membership degrees—*fuzzy partial sets*. The main idea is to replace undefined membership values by a dummy element $*$ that stands outside the scale for truth values L and is incomparable with any $a \in L$. Consequently, original partial membership functions to L of fuzzy sets (with undefined membership values) are replaced by total functions to the extended scale $L \cup \{*\}$ that represent fuzzy partial sets. Next, these membership functions are conveniently manipulated with $*$-extended operations based on the connectives and quantifiers of a suitable chosen fuzzy logic. Note that in this setting, the dummy element $*$ can represent specific types of undefinedness characterized as, e.g., valueless or nonsensical, but not as unknown or yet undefined. Formally, fuzzy partial sets can be handled within the semantic framework of a first-order extension of the recently proposed fuzzy partial propositional logic [3].

In this presentation, we review the semantics of a simple first-order extension of fuzzy partial propositional logic and a simple theory of fuzzy partial sets of the first order introduced in [2]. In a manner similar to [4, Ch. 18] or [1], we introduce a selection of basic notions of fuzzy partial set theory, discuss their variants, and present results on the properties of fuzzy partial class operations and relations.

References

[1] Libor Běhounek and Petr Cintula. Fuzzy class theory. *Fuzzy Sets and Systems*, 154(1):34–55, 2005.

[2] Libor Běhounek and Martina Daňková. Towards fuzzy partial set theory. In J.P. Carvalho et al. (Eds.), *proceedings of IPMU 2016, part II*, volume 611 of *Communications in Computer and Information Science*, pages 482–494. Springer, 2016.

[3] Libor Běhounek and Vilém Novák. Towards fuzzy partial logic. In *Proceedings of the IEEE 45th International Symposium on Multiple-Valued Logics (ISMVL 2015)*, pages 139–144, 2015.

[4] Siegfried Gottwald. *A Treatise on Many-Valued Logics*, volume 9 of *Studies in Logic and Computation*. Research Studies Press, Baldock, 2001.

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First Steps Towards Fuzzy Type Theory with Partial Functions

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When applying type theory to the logical analysis of natural language, we often meet the requirement to deal with partial functions. However, several problems arise when trying to include the latter in the formalism of type theory. One of the main ones is failure of the λ-conversion which significantly reduces the power of the resulting theory.

This paper is a study of fuzzy type theory (FTT) with partial functions. Out of several possibilities we decided to introduce a special value "$*$" which represents "undefined". In the interpretation of FTT, this value lays outside of the corresponding domain.

The value $*$ can be introduced not only among truth values (type o) but also among elements of other types distinct from o. Namely, the value $*_o$ of type o is defined as the formula $\iota_{o(oo)} \cdot \lambda x_o \bot$ which means application of the description operator to the empty set. Similarly, the $*_\epsilon$ is defined as $\iota_{\epsilon(o\epsilon)} \cdot \lambda x_\epsilon *_o$, i.e., the description operator is applied to a fuzzy set on M_ϵ whose membership function is nowhere defined. Similarly, we define

$$*_{\beta\alpha} \equiv \lambda x_\alpha *_\beta$$

which means that "undefined" for more complex types is a nowhere defined function from the set M_α of type α to a set M_β of type β.

In the development of FTT with partial functions, we must be careful because the value "undefined" is a well formed formula. The outcome is that $T \vdash A_o \equiv *_o$ means that the formula A_o is in the theory T equal to "undefined" and so, A_o is also undefined; we formally write $T \vdash ?A_o$. Consequently, a formula A_o is defined if $T \vdash \neg(A_o \equiv *_o)$. In symbols, we write the latter as $T \vdash !A_o$. The predicates "?" and "!" can also be extended to formulas of higher types.

Important outcome of our approach is that the λ-conversion is preserved which makes our system of FTT very powerful. Among many results, we show that $T \vdash *_o$ implies that T is contradictory. We can also introduce definition of partial functions on all types. Namely we can introduce the following predicates:

- *Total function* $\mathrm{TotF}_{o(\beta\alpha)} \equiv \lambda f_{\beta\alpha} \cdot (\forall x_\alpha)(!x_\alpha \Rightarrow f_{\beta\alpha} x_\alpha)$,
- *Partial function* $\mathrm{PartF}_{o(\beta\alpha)} \equiv \lambda f_{\beta\alpha} \cdot (\exists x_\alpha)(!x_\alpha \& f_{\beta\alpha} x_\alpha)$

where $!x_\alpha$ means that the variable x_α represents a defined formula (of type α). A partial function $f_{\beta\alpha}$ is *strict* in T if $T \vdash f_{\beta\alpha} *_\alpha \equiv *_\beta$. It is *non-strict* if $T \vdash !f_{\beta\alpha} *_\alpha$.

Finally, we prove that any consistent theory of FTT with partial functions has a model.

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References

[1] P. Andrews, *An Introduction to Mathematical Logic and Type Theory: To Truth Through Proof*, Kluwer, Dordrecht, 2002.

[2] L. Bhounek, V. Novk: *Towards fuzzy partial logic.* In: Proceedings of the IEEE 45th Intl. Symposium on Multiple-Valued Logics (ISMVL 2015), pp. 139-144, 2015

[3] M. Duží, B. Jespersen, P. Materna, *Procedural Semantics for Hyperintensional Logic*, Springer, Dordrecht, 2010.

[4] W. M. Farmer, *A partial functions version of Church's simple theory of types*, Journal of Symbolic Logic 55 (1990) 1269–1291.

[5] S. Lappiere, *A functional partial semantics for intensional logic,* Notre Dame Journal of Formal Logic 33 (1992) 517–541.

[6] F. Lepage, *Partial functions in type theory*, Notre Dame Journal of Formal Logic 33 (1992) 493–516.

[7] V. Novák, *On fuzzy type theory*, Fuzzy Sets and Systems 149 (2005) 235–273.

[8] V. Novák, *EQ-algebra-based fuzzy type theory and its extensions*, Logic Journal of the IGPL 19 (2011) 512–542.

Dealing with Undefined Values in Fuzzy Partial Logic

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Real-world applications come with the need to handle not only vague information, for which fuzzy logic or fuzzy set theory may be applicable, but also missing or undefined values. Fuzzy logic works with predicates that can have a truth value in a degree, perhaps from the interval $[0, 1]$, to capture a vague truthfulness or falsity. However, in real-world datasets, there are many data values completely unavailable, for some reason. Consider a quality assessment procedure in a factory: the product may be evaluated as OK or faulty, or perhaps some degree of acceptance can be assigned to it. However, some exemplars of the product may not enter the quality process at all – for them, the quality status is *unknown*. Other products may be *inapplicable* for such type of quality assessment, e.g. the quality test of an air condition unit in a car without any such unit is *undefined*. In this paper, we analyze several situations, where the non-existent data may come from, and show within a fuzzy sets framework that different cases of non-existence have to be carefully treated and interpreted in a different way.

The objective of handling missing or undefined values is not new. The fundamental grounds in mathematical logic were established by Kleene, Bochvar, Sobociński and others, who studied the properties of three-valued logics 0/1/*, which was also studied by Łukasiewicz in 1920 in [1]. These authors showed that the third value * may represent an unknown, undefined or indeterminate truth value. An overview of main contributions can be found e.g. in [2].

Their work is being generalized to fuzzy propositional partial logic and later extended to predicate partial fuzzy logic by Novák, Běhounek and Daňková in [3] and [4]. They propose a fuzzy logic that handles a special truth value * with several types of fuzzy logical connectives that each treat the * value in a different way. Inspired by the approach of Sobociński, Kleene and Bochvar, they define three sets of connectives, which differ in handling of * – see Table 1. We recall that this kind of "undefined" truth values differs form "indeterminate" or "underspecified" values which were studied in [6–8].

Recall that the approach introduced in [4] is based on expansions of well-known fuzzy logic MTL_Δ of left-continuous t-norms (see [5]). The mentioned class of fuzzy logics includes most know fuzzy logics containing the connective Δ, such as Łukasiewicz logic, Gödel logic and Product logic, the logics IMTL_Δ, SMTL_Δ and ŁΠ. In [4], the primitive connectives are the truth constant $*$ representing the *undefined*, the unary connective ! for the crisp modality "*is defined*" and the binary connective $\wedge_K$ for the Kleene-style variant of $\wedge$. Applying of these primitive connectives all other connectives $(\otimes, \vee, \rightarrow, \neg, \leftrightarrow)$ can be defined in *Bochvar-style, Kleene-style, Sobociński-style, Mc-Carthy-style.* We recall three types of conjunctions accordingly to Sobociński, Kleene and Bochvar style for handling of "*" in Table 1.

Note that Sobociinski style treats $*$ as an ignorable nonsense: $*$ is ignored, if possible: $* \wedge a = a$. Thus in $\wedge_S$, the $*$ truth value acts as neutral element. On the other hand, Bochvar

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Table 1: Sobociński-, Kleene-, and Bochvar-style conjunctions, as defined in [3, 4]

$\wedge_S$	0	b	1	$*$
0	0	0	0	0
a	0	$a \wedge b$	a	a
1	0	b	1	1
$*$	0	b	1	$*$

$\wedge_K$	0	b	1	$*$
0	0	0	0	0
a	0	$a \wedge b$	a	$*$
1	0	b	1	$*$
$*$	0	$*$	$*$	$*$

$\wedge_B$	0	b	1	$*$
0	0	0	0	$*$
a	0	$a \wedge b$	a	$*$
1	0	b	1	$*$
$*$	$*$	$*$	$*$	$*$

$\wedge_B$ (also called Bochvar-internal [2], or Kleene's weak) acts as annihilator. Kleene's $\wedge_K$ (a.k.a. Kleene's strong).

In this paper, we suggest a different approach: to introduce *several truth values representing different kinds of unavailability together with a single type of fuzzy logical connectives.* As opposed to the former approach, our suggestion requires to study the interaction of different types of unavailability within logical connectives.

References

[1] J. Lukasiewicz: *O logice trojwartosciowej.* Ruch filozoficzny. **5** (1920) 170–171.

[2] G. Malinowski: *The Many Valued and Nonmonotonic Turn in Logic.* Amsterdam, North-Holland. (2007).

[3] L. Běhounek and V. Novák: *Towards fuzzy partial logic.* In Proceedings of the IEEE 45th International Symposium on Multiple-Valued Logics (ISMVL 2015), **5** (2015) 139–144.

[4] L. Běhounek and Daňková, M.: *Towards Fuzzy Partial Set Theory.* Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU 2016), **5** (2016) 482-494.

[5] P. Hájek: *Metamathematics of Fuzzy Logic.*, Dordrecht, Kluwer, (1998).

[6] Esteva, F. and Garcia-Calvés, P. and Godo, L.: *Enriched interval bilattices and partial many-valued logics: An approach to deal with graded truth and imprecision.* International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, **02** (1994) 37-54.

[7] Gasse, B. V. and Cornelis, C. and Deschrijver, G. and Kerre, E.: *A characterization of interval-valued residuated lattices.* International Journal of Approximate Reasoning, **49** (2008) 478-487.

[8] Hájek, P.: *On vagueness, truth values and fuzzy logic.* Studia Logica , **91** (2009) 367-382.

A survey on Coherence Notions useful for Multi-adjoint Normal Logic Programming Theory

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Real-life databases frequently contain inconsistent and contradictory information which can be reliable and useful and so, it is important develop suitable tools to manage and measure such information [2,4–6]. To the best of our knowledge, logical theories have been somewhat removed from achieving this goal, since their strategies have been based on either avoiding/ignoring the conflictive information or try to repair it.

We are interested in choosing a coherence notion suitable to handle inconsistent information included in multi-adjoint normal logic programs. An initial study on the syntax of multi-adjoint normal logic programs and the existence of their minimal models has been introduced in [1], by means of the stable model semantics.

Our contribution will deal with different notions closely related to the concept of coherence such as self-contradiction in fuzzy sets [10], x-consistent interpretation [9], consistency in interlaced bilattices [3] and coherent interpretation [7, 8]. An adaptation of the mentioned notions above to our logic programming framework, together with several properties and remarks, will be presented in order to select the most suitable coherence notion.

References

[1] M. E. Cornejo, D. Lobo, and J. Medina. Towards multi-adjoint logic programming with negations. In L. Koczy and J. Medina, editors, *8th European Symposium on Computational Intelligence and Mathematics (ESCIM 2016)*, pages 24–29, 2016.

[2] D. Dubois, S. Konieczny, and H. Prade. Quasi-possibilistic logic and its measures of information and conflict. *Fundam. Inf.*, 57(2-4):101–125, 2003.

[3] M. Fitting. Bilattices and the semantics of logic programming. *The Journal of Logic Programming*, 11(2):91–116, 1991.

[4] J. Grant and A. Hunter. Measuring inconsistency in knowledgebases. *Journal of Intelligent Information Systems*, 27(2):159–184, 2006.

[5] S. Jabbour and B. Raddaoui. *Measuring Inconsistency through Minimal Proofs*, pages 290–301. Springer Berlin Heidelberg, Berlin, Heidelberg, 2013.

[6] K. Knight. Measuring inconsistency. *Journal of Philosophical Logic*, 31(1):77–98, 2002.

[7] N. Madrid and M. Ojeda-Aciego. Measuring inconsistency in fuzzy answer set semantics. *IEEE Transactions on Fuzzy Systems*, 19(4):605–622, 2011.

Acknowledgement Partially supported by the State Research Agency (AEI) and the European Regional Development Fund (ERDF) project TIN2016-76653-P.

[8] N. Madrid and M. Ojeda-Aciego. On the measure of incoherent information in extended multi-adjoint logic programs. In *2013 IEEE Symposium on Foundations of Computational Intelligence (FOCI)*. Institute of Electrical and Electronics Engineers (IEEE), 2013.

[9] D. V. Nieuwenborgh, M. D. Cock, and D. Vermeir. An introduction to fuzzy answer set programming. *Annals of Mathematics and Artificial Intelligence*, 50(3-4):363–388, 2007.

[10] E. Trillas, C. Alsina, and J. Jacas. On contradiction in fuzzy logic. *Soft Computing - A Fusion of Foundations, Methodologies and Applications*, 3(4):197–199, 1999.

On the relationship of the f-index of contradiction with the f-index of inclusion

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In crisp logic the contradiction is defined as the incompatibility between two propositions. Formally, we can say that a predicate symbol P is contradictory with respect to another Q if and only if $(\nexists t)P(t)\wedge Q(t)$ or equivalently, that $(\forall t)P(t) \to \neg Q(t)$. When we move to the Fuzzy Logic paradigm, the satisfiability of such formulas is graded and therefore, we can define *degrees of contradiction*. The most known degree of contradiction between fuzzy predicate symbols P and Q refers to the former formula and is defined as the supremum of the set $\{P(t) \wedge Q(t) \mid t \in T\}$, where T is the set of terms and $\wedge$ is conveniently represented by a fuzzy conjunction (e.g. a t-norm). Similarly, we can define the degree of contradiction between P and Q by following the formula $(\forall t)P(t) \to \neg Q(t)$; i.e., as the infimum of the set $\{P(t) \to \neg Q(t) \mid t \in T\}$, where $\to$ is conveniently represented by a fuzzy implication (e.g., a residuated implication). It is noticeable the following difference between both formulae: the former formula is based on the idea *P and Q cannot be true at the same time* whereas the latter describes the idea *a high degree of truth in P implies a low degree of truth in Q*. Although in crisp logic both interpretations are equivalent, in fuzzy logic not. One common shortcoming of the previous two formulae is related to the choice of the connectives $\wedge$ and $\to$. To avoid such election, we define in [1] the f-index of contradiction which represents the contradiction between two prepositional symbols by means of a function instead of a number in the unit interval $[0, 1]$.

On the other hand, in the fuzzy set paradigm, given a fuzzy implication $\to$, the degree of inclusion of a fuzzy set P in Q is commonly defined as the infimum of the set $\{P(t) \to Q(t) \mid t \in T\}$. Thus the formula $(\forall t)P(t) \to \neg Q(t)$ used to characterize the contradiction can be understood as *$P(t)$ is included in the complement of Q*; considering P and Q as (fuzzy) sets. Noting the similarity of this degree of inclusion with the degree of contradiction, we introduced in [2] the f-index of inclusion based on the idea followed to define the f-index of contradiction. In this talk we recall both notions, i.e. f-indexes of contradiction and inclusion, and show how both are related.

References

[1] H. Bustince, N. Madrid, and M. Ojeda-Aciego. A Measure of Contradiction Based on the Notion of N-Weak-Contradiction. In *IEEE Intl Conf on Fuzzy Systems (FUZZ-IEEE'13)*, pages 1–6, 2013.

[2] N. Madrid, M. Ojeda-Aciego, and I. Perfilieva. f-inclusion indexes between fuzzy sets. In *Proc. of IFSA-EUSFLAT*, 2015.

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