Reasoning with Qualitative Velocity. Towards a Hybrid Approach*

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Abstract. Qualitative description of the movement of objects can be very important when there are large quantity of data or incomplete information, such as in positioning technologies and movement of robots. We present a first step in the combination of fuzzy qualitative reasoning and quantitative data obtained by human interaction and external devices as GPS, in order to update and correct the qualitative information. We consider a Propositional Dynamic Logic which deals with qualitative velocity and enables us to represent some reasoning tasks about qualitative properties. The use of logic provides a general framework which improves the capacity of reasoning. This way, we can infer additional information by using axioms and the logic apparatus. In this paper we present sound and complete relational dual tableau that can be used for verification of validity of formulas of the logic in question.

1 Introduction

Qualitative reasoning, QR, is an area of AI which tries to simulate the way of humans think in almost all situations. For example, we do not need to know the exact value of velocity and position of a car in order to drive it. When raising or answering questions about moving objects, both qualitative and quantitative responses are possible, as stated in [9]. However, human beings usually prefer to communicate using qualitative information according to their intuition rather than using quantitative values. Moreover, representing and reasoning with quantitative information can lead to information overload, that is, there is more information to be handled than the one that can be processed. A form of QR is order of magnitude reasoning, where the values are represented by different qualitative classes. For example, talking about velocity we may consider *slow*, *normal*, and *quick* as qualitative classes.

As said in [27], qualitative models can be seen as discrete abstractions of continuous and hybrid systems and can be fully explored by a verification tool providing conservative analysis of hybrid systems.

The use of logic in QR, as in other areas of AI, provides a general framework which allows us to improve the capacity of solving problems and, as we will see in this paper, to deal with the reasoning problem. This way, we can infer additional information by using axioms and the logic apparatus. There are several applications of logics for QR (see e.g., [2, 10]) and many of them concern spatial reasoning. As an example of logic for order of magnitude reasoning, see [4]; a theorem prover for one of these logics can be seen in [16]; and some implementations in [5, 15].

Qualitative description of the movement of objects can be very important when there are large quantity of data, such as in positioning technologies (GPS, wireless communication) and

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movement of robots. In this direction, the concept of qualitative velocity [11, 28], together with qualitative distance and orientation, could help in order to represent spatial reasoning.

In a hybrid intelligence system, multiple techniques are used in order to obtain an efficient solution for a particular problem [1, 7, 8, 14, 23]. In our case, this combination is made by using qualitative reasoning, fuzzy aspects as in [3], and quantitative data obtained by human interaction and external devices as GPS, in order to update and correct the qualitative information. There are recent papers where similar combinations have been presented. For instance, in [26] a quantitative method is used for analyzing and comparing trajectories of robots using point distribution models; in [17] a simulator that combines qualitative reasoning, a geographic information system and targeted probabilistic calculations is presented; and in [31] a mix of qualitative and quantitative data is used for a hybrid modeling approach is studied.

Some papers [6, 18, 19] are developing the qualitative kinematics models studied in [12, 13, 21]. The relative movement of one object with respect to another has been studied by the Region Connection Calculus [24] and the Qualitative Trajectory Calculus [9, 30]. However, as far as we know, the first paper which proposes a logic framework for qualitative velocity is [3], where a Propositional Dynamic Logic for order of magnitude reasoning to deal with the concept of qualitative velocity is proposed. The main advantages of this approach are: the possibility of constructing complex relations from simpler ones; the flexibility for using different levels of granularity; its possible extension by adding other spatial components, such as position, distance, cardinal directions, etc.; the use of a language close to programming languages; and, above all, the strong support of logic in spatial reasoning. Following [11], velocity of an object B with respect to another object A is represented by two components: module and orientation, each one given by a qualitative class. If we consider a velocity of B with respect to A, and another velocity of C with respect to B, the composition of these two velocities consists of obtaining the velocity of C with respect to A. For example, if (Q,l) represents a *quick* velocity towards the *left* orientation of B with respect to A, and (N,r) is a *normal* velocity towards the *right* of C with respect to B, the composition is a velocity of C with respect to A, that could be either (Q,l) or (N,l), that is, a *quick* or *normal* velocity towards the left orientation. The results of these compositions could depend on the specific problem we are dealing with. In the following section, we consider the logic QV where some assumptions about these compositions are posed in its models.

In this paper we present a first step in the construction of an hybrid approach to deal with qualitative velocity. We show a sound and complete relational dual tableau for the Propositional Dynamic Logic of qualitative velocity introduced in [3], which can be used for verification of validity of its formulas. The system is based on Rasiowa-Sikorski diagrams for first-order logic [25]. The common language of most of relational dual tableaux is the logic of binary relations, which is a logical counterpart to the class RRA of (representable) relation algebras introduced by Tarski in [29]. The formulas of the classical logic of binary relations are intended to represent statements saying that two objects are related. Relations are specified in the form of relational terms. Terms are built from relational variables and/or relational constants with relational operations of union, intersection, complement, composition, and converse.

Relational dual tableaux are powerful tools for verification of validity as well as for proving entailment, finite model checking (i.e., verification of truth of a statement in a particular fixed finite model) and finite satisfaction (i.e., verification that a statement is satisfied by some fixed objects of a finite model). A comprehensive survey on applications of dual tableaux methodology to various theories and logics can be found in [22]. The main advantage of relational methodology is the possibility of representation within a uniform formalism the three basic components of formal systems: syntax, semantics, and deduction apparatus. Hence, the relational approach provides a general framework for representation, investigation and implementation of theories with different languages and/or semantics.

The paper is organized as follows. In Section 2 we present the Propositional Dynamic Logic of qualitative velocity, QV, its syntax and semantics. Relational formalization of the logic is presented in Section 3. In Section 4 we present the relational dual tableau for this logic, and we study its soundness and completeness; moreover, we show an example of the relational proof of validity of a QV-formula. Conclusions and final remarks are discussed in Section 5.

2 Logic QV for reasoning with qualitative velocity

In this section we present the syntax and semantics of the logic QV for order of magnitude qualitative reasoning to deal with the concept of qualitative velocity. We consider the set of qualitative velocities $L_1 = \{\mathsf{z}, \mathsf{v}_1, \mathsf{v}_2, \mathsf{v}_3\}$, where $\mathsf{z}, \mathsf{v}_1, \mathsf{v}_2, \mathsf{v}_3$ represent zero, slow, normal, and quick velocity, respectively; and the set of qualitative orientations $L_2 = \{\mathsf{n}, \mathsf{o}_1, \mathsf{o}_2, \mathsf{o}_3, \mathsf{o}_4\}$ representing none, front, right, back, and left orientations, respectively. Thus, we consider four qualitative classes for the module of the velocities, and five qualitative classes for the orientation of the velocity. Orientations o_j and $\mathsf{o}_{\mathsf{j}+2}$, for $\mathsf{j} \in \{1, 2\}$, are interpreted as *opposite*. Furthermore, orientations o_j and $\mathsf{o}_{\mathsf{j}+1}$, for $\mathsf{j} \in \{1, 2, 3\}$, are interpreted as *perpendicular*.

The logic QV is an extension of Propositional Dynamic Logic PDL which is a framework for specification and verification of dynamic properties of systems. It is a multimodal logic with the modal operations of necessity and possibility determined by binary relations understood as state transition relations or input-output relations associated with computer programs. The vocabulary of the language of QV consists of symbols from the following pairwise disjoint sets: $\mathbb{V}$ - a countably infinite set of propositional variables; $\mathbb{C} = L_1 \times L_2$ - the set of constants representing labels from the set $L_1 \times L_2$; $\mathbb{SP} = \{\otimes_\star | \star \in \mathbb{C}\}$ - the set of relational constants representing specific programs; $\{\cup, ;, ?, ^*\}$ - the set of relational operations, where $\cup$ is interpreted as a nondeterministic choice, ; is interpreted as a sequential composition of programs, ? is the test operation, and * is interpreted as a nondeterministic iteration; $\{\neg, \vee, \wedge, \rightarrow, [\,], \langle\rangle\}$ - the set of propositional operations of negation, disjunction, conjunction, implication, necessity, and possibility, respectively.

The set of QV-relational terms interpreted as compound programs and the set of QV-formulas are the smallest sets containing $\mathbb{SP}$ and $\mathbb{V} \cup \mathbb{C}$, respectively, and satisfying the following conditions:

– If S and T are QV-relational terms, then so are $S \cup T$, $S\,;T$, and T^*.
– If φ is a QV-formula, then $\varphi?$ is a QV-relational term.
– If φ and ψ are QV-formulas, then so are $\neg\varphi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, and $\varphi \rightarrow \psi$.
– If φ is a QV-formula and T is a QV-relational term, then $[T]\varphi$ and $\langle T\rangle\varphi$ are QV-formulas.

Given a binary relation R on a set W and $X \subseteq W$, we define:

$$R(X) \stackrel{\text{df}}{=} \{w \in W \mid \exists x \in X,\ (x, w) \in R\}.$$

Fact 1 *For every binary relation R on a set W and for all $X, Y \subseteq W$:*

$$R(X) \subseteq Y \textit{ iff } (R^{-1}\,; (X \times W)) \subseteq (Y \times W).$$

A QV-*model* is a structure $\mathcal{M} = (W, m)$, where W is a non-empty set of states and m is a meaning function satisfying the following conditions:

- $W = \bigcup_{\star \in \mathbb{C}} \star$ where all $\star$'s are pairwise disjoint subsets of states understood as states of objects affected by a qualitative velocity
- $m(p) \subseteq W$ for every $p \in \mathbb{V}$
- $m(\star) = \star$, for every $\star \in \mathbb{C}$
- $m(\otimes_\star) \subseteq W \times W$, for every $\otimes_\star \in \mathbb{SP}$, and, in addition, for all $\mathsf{v}, \mathsf{v_r}, \mathsf{v_s} \in L_1$ and for all $\mathsf{o}, \mathsf{o_j}, \mathsf{o_{j+1}}, \mathsf{o_{j+2}} \in L_2$, the following hold:
  (S1) $m(\otimes_{(\mathsf{v},\mathsf{o})})\,;m(\otimes_{(\mathsf{z},\mathsf{n})}) = m(\otimes_{(\mathsf{v},\mathsf{o})})$
  (S2) $m(\otimes_{(\mathsf{v},\mathsf{o_j})})\,;m(\otimes_{(\mathsf{v},\mathsf{o_{j+2}})}) = m(\otimes_{(\mathsf{z},\mathsf{n})})$, for $\mathsf{j} \in \{1, 2\}$
  (S3) $m(\otimes_{(\mathsf{v},\mathsf{o_{j+1}})})(m(\mathsf{v}, \mathsf{o_j})) \subseteq m(\mathsf{v}, \mathsf{o_j}) \cup m(\mathsf{v}, \mathsf{o_{j+1}})$, for $\mathsf{j} \in \{1, 2, 3\}$
  (S4) $m(\otimes_{(\mathsf{v_s},\mathsf{o_{j+1}})})(m(\mathsf{v_r}, \mathsf{o_j})) \subseteq m(\mathsf{v_s}, \mathsf{o_{j+1}})$, for $\mathsf{j} \in \{1, 2, 3\}$ and $\mathsf{r} < \mathsf{s}$
  (S5) $m(\otimes_{(\mathsf{v_s},\mathsf{o})})(m(\mathsf{v_r}, \mathsf{o})) \subseteq m(\mathsf{v_s}, \mathsf{o}) \cup m(\mathsf{v_3}, \mathsf{o})$, for $\mathsf{s} \in \{2, 3\}$ and $\mathsf{r} < \mathsf{s}$
  (S6) $m(\otimes_{(\mathsf{v_s},\mathsf{o_{j+2}})})(m(\mathsf{v_r}, \mathsf{o_j})) \subseteq m(\mathsf{v_s}, \mathsf{o_{j+2}}) \cup m(\mathsf{v_{s-1}}, \mathsf{o_{j+2}})$, for $\mathsf{j} \in \{1, 2\}$, $\mathsf{s} \in \{2, 3\}$, and $\mathsf{r} < \mathsf{s}$

m extends to all the compound QV-relational terms and formulas:

- $m(T^*) = m(T)^* = \bigcup_{i \geq 0} m(T^i)$, where T^0 is the identity relation on W and $T^{i+1} \stackrel{\text{df}}{=} (T^i\,; T)$
- $m(S \cup T) = m(S) \cup m(T)$
- $m(S\,; T) = m(S)\,; m(T)$
- $m(\varphi?) = \{(s, s) \in W \times W : s \in m(\varphi)\}$
- $m(\neg\varphi) = W \setminus m(\varphi)$
- $m(\varphi \vee \psi) = m(\varphi) \cup m(\psi))$
- $m(\varphi \wedge \psi) = m(\varphi) \cap m(\psi))$
- $m(\varphi \rightarrow \psi) = m(\neg\varphi) \cup m(\psi))$
- $m([T]\varphi) = \{s \in W |\text{ for all } t \in W, \text{ if } (s, t) \in m(T), \text{ then } t \in m(\varphi)\}$
- $m(\langle T \rangle\varphi) = \{s \in W |\text{ exists } t \in W \text{ such that } (s, t) \in m(T) \text{ and } t \in m(\varphi)\}$

Given a QV-model $\mathcal{M} = (W, m)$, a QV-formula φ is said to be *satisfied in* $\mathcal{M}$ *by* $s \in W$, $\mathcal{M}, s \models \varphi$ for short, whenever $s \in m(\varphi)$. As usual, a formula is true in a model whenever it is satisfied in all states of the model and it is QV-valid iff it is true in all QV-models.

Intuitively, $(s, s') \in m(T)$ means that there exists a computation of program T starting in the state s and terminating in the state s'. Program $S \cup T$ performs S or T nondeterministically; program $S\,; T$ performs first S and then T. Expression $\varphi?$ is a command to continue if φ is true, and fail otherwise. Program T^* performs T zero or more times sequentially. For example, the formula $\langle(\mathsf{v_1}, \mathsf{o_4})?\rangle\varphi$ is satisfied in s whenever s is a slow velocity towards the left orientation and φ is satisfied in s; the formula $[\otimes^*_{(\mathsf{v_3},\mathsf{o_2})}]\varphi$ is satisfied in s iff for every velocity s' obtained by the repetition of the composition of s with a quick velocity towards the right orientation a nondeterministically chosen finite number of times, φ is satisfied in s'; the formula $[\otimes_{(\mathsf{v_1},\mathsf{o_4})}\,; \otimes_{(\mathsf{v_2},\mathsf{o_2})}]\varphi$ is satisfied in s iff for every velocity s' obtained by composing s with a slow velocity towards the left followed by a normal velocity towards the right orientation, φ is satisfied in s'.

Example 1. Let us consider the case study of ball interception of simulated soccer agents, presented in [3]. Suppose that the ball is located at a point B and is moving with a velocity $(\mathsf{v_b}, \mathsf{o_b})$ and the robot is at point R and it is not moving at this instant (see Figure 1). Suppose also that the robot can calculate the qualitative velocity needed to catch the ball at the current instant and position and this velocity is $(\mathsf{v_c}, \mathsf{o_c})$. A simple vectorial argument leads us to the fact that the composition of both velocities has to be the velocity needed to catch the ball. This condition can be expressed in our language by the formula $(\mathsf{z}, \mathsf{n}) \rightarrow [\otimes_{(\mathsf{v_c},\mathsf{o_c})}; \otimes_{(\mathsf{v_b},\mathsf{o_b})}]\mathsf{OK}$, where (z, n) means that the robot is not moving at this instant, and OK means that the velocity of the robot is the correct one in order to catch the ball. the validity of this formula has to be checked as the robot is moving towards the ball and has to be corrected if it is not OK. The correction of this movement could require the human intervention and, on the other hand, the position of the robot may need some external device as a GPS.

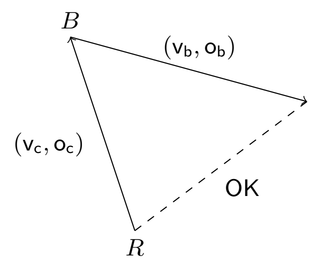

Fig. 1. Catching a ball when the robot is not moving

3 Relational representation of logic QV

In this section we present the relational formalization of logic QV providing a framework for deduction in logic QV. First, we define the relational logic $\mathsf{RL_{QV}}$ appropriate for expressing QV-terms and QV-formulas. Then, we translate all QV-terms and QV-formulas into relational terms and we show the equivalence of validity between a modal formula and its corresponding relational formula. The vocabulary of the language of the relational logic $\mathsf{RL_{QV}}$ consists of of symbols from the following pairwise disjoint sets: $\mathbb{OV} = \{x, y, z, \ldots\}$ - a countably infinite set of object variables; $\mathbb{RV} = \{P, Q, \ldots\}$ - a countably infinite set of binary relational variables; $\mathbb{RC} = \{1, 1'\} \cup \{R_\star, \Psi_\star | \star \in \mathbb{C}\}$ - the set of relational constants, where $\mathbb{C}$ is defined as in QV-models; $\mathbb{OP} = \{-, \cup, \cap, ;, ^{-1}, ^*\}$ - the set of relational operation symbols. The intuitive meaning of the relational representation of the symbols of logic QV is as follows: propositional variables are represented by relational variables; constants from $\mathbb{C}$ are represented by relational constants $\Psi_\star$ interpreted as right ideal binary relations; relational constants $R_\star$ correspond to specific programs $\otimes_\star$; the relational constants 1 (the universal relation), $1'$ (the identity relation), and relational operations are used in the representation of compound QV-formulas.

The set of $\mathsf{RL_{QV}}$-terms is the smallest set containing relational variables and relational constants and closed on all the relational operations. $\mathsf{RL_{QV}}$-formulas are of the form xTy, where T is an $\mathsf{RL_{QV}}$-relational term and x, y are object variables. An $\mathsf{RL_{QV}}$-model is a structure $\mathcal{M} = (W, m)$, where W is defined as in QV-models and m is the meaning function that satisfies:

- $m(P) \subseteq W \times W$, for every $P \in \mathbb{RV} \cup \{R_\star | \star \in \mathbb{C}\}$
- $m(\Psi_\star) = \star \times W$, for every $\star \in \mathbb{C}$

- $m(1')$ is an equivalence relation on W
- $m(1')\,;m(P) = m(P)\,;m(1') = m(P)$, for every $P \in \mathbb{RV} \cup \mathbb{RC}$ (the extensionality property)
- $m(1) = W \times W$
- For all $\mathsf{v}, \mathsf{v_r}, \mathsf{v_s} \in L_1$ and for all $\mathsf{o}, \mathsf{o_j}, \mathsf{o_{j+1}}, \mathsf{o_{j+2}} \in L_2$, the following hold:

(RS1) $m(R_{(\mathsf{v},\mathsf{o})})\,; m(R_{(\mathsf{z},\mathsf{n})}) = m(R_{(\mathsf{v},\mathsf{o})})$
(RS2) $m(R_{(\mathsf{v},\mathsf{o_j})})\,; m(R_{(\mathsf{v},\mathsf{o_{j+2}})}) = m(R_{(\mathsf{z},\mathsf{n})})$, for $\mathsf{j} \in \{1, 2\}$
(RS3) $m(R_{(\mathsf{v},\mathsf{o_{j+1}})})^{-1}\,; m(\Psi_{(\mathsf{v},\mathsf{o_j})}) \subseteq m(\Psi_{(\mathsf{v},\mathsf{o_j})}) \cup m(\Psi_{(\mathsf{v},\mathsf{o_{j+1}})})$, for $\mathsf{j} \in \{1, 2, 3\}$
(RS4) $m(R_{(\mathsf{v_s},\mathsf{o_{j+1}})})^{-1}\,; m(\Psi_{(\mathsf{v_r},\mathsf{o_j})}) \subseteq m(\Psi_{(\mathsf{v_s},\mathsf{o_{j+1}})})$, for $\mathsf{j} \in \{1, 2, 3\}$ and $\mathsf{r} < \mathsf{s}$
(RS5) $m(R_{(\mathsf{v_s},\mathsf{o})})^{-1}\,; m(\Psi_{(\mathsf{v_r},\mathsf{o})}) \subseteq m(\Psi_{(\mathsf{v_s},\mathsf{o})}) \cup m(\Psi_{(\mathsf{v_3},\mathsf{o})})$, for $\mathsf{s} \in \{2, 3\}$ and $\mathsf{r} < \mathsf{s}$
(RS6) $m(R_{(\mathsf{v_s},\mathsf{o_{j+2}})})^{-1}\,; m(\Psi_{(\mathsf{v_r},\mathsf{o_j})}) \subseteq m(\Psi_{(\mathsf{v_s},\mathsf{o_{j+2}})}) \cup m(\Psi_{(\mathsf{v_{s-1}},\mathsf{o_{j+2}})})$, for $\mathsf{j} \in \{1, 2\}$, $\mathsf{s} \in \{2, 3\}$, and $\mathsf{r} < \mathsf{s}$

- m extends to all the compound relational terms as follows:

$$m(-T) = m(1) \cap -m(T), \qquad m(T^{-1}) = m(T)^{-1},$$
$$m(S \cup T) = m(S) \cup m(T), \qquad m(S\,;T) = m(S)\,;m(T),$$
$$m(S \cap T) = m(S) \cap m(T), \qquad m(T^*) = m(T)^*.$$

Observe that the conditions (RS1), ..., (RS6) are relational counterparts of the conditions (S1), ..., (S6) assumed in QV-models. An $\mathsf{RL_{QV}}$-model $\mathcal{M}$ in which $1'$ is interpreted as the identity is said to be *standard*. Let $v\colon \mathbb{OV} \to W$ be a valuation in an $\mathsf{RL_{QV}}$-model $\mathcal{M}$. An $\mathsf{RL_{QV}}$-formula xTy is said to be satisfied in $\mathcal{M}$ by v whenever $(v(x), v(y)) \in m(T)$. A formula φ is true in $\mathcal{M}$ iff it is satisfied in $\mathcal{M}$ by all the valuations and it is $\mathsf{RL_{QV}}$-valid whenever it is true in all $\mathsf{RL_{QV}}$-models.

Now, we define the translation τ of QV-terms and QV-formulas into $\mathsf{RL_{QV}}$-relational terms. Let τ' be a one-to-one mapping that assigns relational variables to propositional variables. The translation τ is defined as follows:

- $\tau(p) = (\tau'(p)\,;1)$, for every $p \in \mathbb{V}$
- $\tau(\star) = \Psi_\star$, for every $\star \in \mathbb{C}$
- $\tau(\otimes_\star) = R_\star$, for every $\otimes_\star \in \mathbb{SP}$

For all relational terms T and S:

- $\tau(T^*) = \tau(T)^*$
- $\tau(S \cup T) = \tau(S) \cup \tau(T)$
- $\tau(S\,;T) = \tau(S)\,;\tau(T)$
- $\tau(\varphi?) = 1' \cap \tau(\varphi)$
- $\tau(\varphi \vee \psi) = \tau(\varphi) \cup \tau(\psi)$
- $\tau(\varphi \wedge \psi) = \tau(\varphi) \cap \tau(\psi)$
- $\tau(\varphi \to \psi) = \tau(\neg\varphi \vee \psi)$
- $\tau(\langle T\rangle\varphi) = \tau(T)\,;\tau(\varphi)$
- $\tau([T]\varphi) = -(\tau(T)\,;-\tau(\varphi))$.

Relational terms obtained from formulas of logic QV include both declarative information and procedural information provided by these formulas. The declarative part which represents static facts about a domain is represented by means of a Boolean reduct of algebras of relations, and the procedural part, which is intended to model dynamics of the domain, requires the relational operations. In the relational terms which represent the formulas after the translation, the two types of information receive a uniform representation and the process of reasoning about both statics and dynamics, and about relationships between them can be performed within the framework of a single uniform formalism.

Theorem 1.
For every QV*-formula* φ *and for all object variables* x *and* y*, the following conditions are equivalent:*

1. φ *is* QV*-valid.*
2. $x\tau(\varphi)y$ *is true in all standard* $\mathsf{RL_{QV}}$*-models.*

4 Relational dual tableau for QV

In this section we present a dual tableau for the logic $\mathsf{RL_{QV}}$ that can be used for verification of validity of QV-formulas. Relational dual tableaux are determined by the axiomatic sets of formulas and rules which apply to finite sets of relational formulas. The axiomatic sets take the place of axioms. The rules are intended to reflect properties of relational operations and constants. There are two groups of rules: decomposition rules and specific rules. Although most often the rules of dual tableaux are finitary, the dual tableau system for logic QV includes an infinitary rule reflecting the behaviour of an iteration operation. Given a formula, the decomposition rules of the system enable us to transform it into simpler formulas, or the specific rules enable us to replace a formula by some other formulas. The rules have the following general form:

$$\text{(rule)} \qquad \frac{\Phi(\overline{x})}{\Phi_1(\overline{x}_1, \overline{u}_1, \overline{w}_1) \mid \ldots \mid \Phi_n(\overline{x}_j, \overline{u}_j, \overline{w}_j) \mid \ldots}$$

where $j \in J$, for some (possibly infinite) set J, $\Phi(\overline{x})$ is a finite (possibly empty) set of formulas whose object variables are among the elements of set$(\overline{x})$, where $\overline{x}$ is a finite sequence of object variables and set$(\overline{x})$ is a set of elements of sequence $\overline{x}$; every $\Phi_j(\overline{x}_j, \overline{u}_j, \overline{w}_j)$, $j \in J$, is a finite non-empty set of formulas, whose object variables are among the elements of set$(\overline{x}_j)\cup$set$(\overline{u}_j)\cup$ set$(\overline{w}_j)$, where $\overline{x}_j, \overline{u}_j, \overline{w}_j$ are finite sequences of object variables such that set$(\overline{x}_j) \subseteq$ set$(\overline{x})$, set$(\overline{u}_j)$ consists of the object variables that may be instantiated to arbitrary object variables when the rule is applied (usually to the object variables that appear in the set to which the rule is being applied), set$(\overline{w}_j)$ consists of the object variables that must be instantiated to pairwise distinct new variables (not appearing in the set to which the rule is being applied) and distinct from any variable of sequence $\overline{u}_j$. A rule of the form (rule) is *applicable* to a finite set X of formulas whenever $\Phi(\overline{x}) \subseteq X$. As a result of an application of a rule of the form (rule) to set X, we obtain the sets $(X \setminus \Phi(\overline{x})) \cup \Phi_j(\overline{x}_j, \overline{u}_j, \overline{w}_j)$, for every $j \in J$. A set to which a rule is applied is called the *premise* of the rule, and the sets obtained by the application of the rule are called its *conclusions*. If the set J is finite, then a rule of the form (rule) is said to be *finitary*, otherwise it is referred to as *infinitary*. Thus, if J has n elements, then the rule of the form (rule) has n conclusions.

A finite set $\{\varphi_1, \ldots, \varphi_n\}$ of $\mathsf{RL_{QV}}$-formulas is said to be an $\mathsf{RL_{QV}}$*-set* whenever for every $\mathsf{RL_{QV}}$-model $\mathcal{M}$ and for every valuation v in $\mathcal{M}$ there exists $i \in \{1, \ldots, n\}$ such that φ_i is satisfied by v in $\mathcal{M}$. It follows that the first-order disjunction of all the formulas from an $\mathsf{RL_{QV}}$-set is valid in the first-order logic. A rule of the form (rule) is $\mathsf{RL_{QV}}$*-correct* whenever for every finite set X of $\mathsf{RL_{QV}}$-formulas, $X \cup \Phi(\overline{x})$ is an $\mathsf{RL_{QV}}$-set if and only if $X \cup \Phi_j(\overline{x}_j, \overline{u}_j, \overline{w}_j)$ is an $\mathsf{RL_{QV}}$-set, for every $j \in J$, i.e., the rule preserves and reflects validity. It follows that ',' (comma) in the rules is interpreted as disjunction and '|' (branching) is interpreted as conjunction.

$\mathsf{RL_{QV}}$-dual tableau includes decomposition rules of the following forms, for any object variables x and y and for any relational terms S and T:

$(\cup)\ \dfrac{x(S\cup T)y}{xSy, xTy}$ $\quad (-\cup)\ \dfrac{x-(S\cup T)y}{x-Sy \mid x-Ty}$ $\quad (\cap)\ \dfrac{x(S\cap T)y}{xSy \mid xTy}$ $\quad (-\cap)\ \dfrac{x-(S\cap T)y}{x-Sy, x-Ty}$

$(-)\ \dfrac{x--Ty}{xTy}$ $\quad (^{-1})\ \dfrac{xT^{-1}y}{yTx}$ $\quad (-^{-1})\ \dfrac{x-T^{-1}y}{y-Tx}$

$(;)\ \dfrac{x(S\,;T)y}{xSz, x(S\,;T)y \mid zTy, x(S\,;T)y}$ for any object variable z $\quad (-;)\ \dfrac{x-(S\,;T)y}{x-Sz, z-Ty}$ for a new object variable z

$(^*)\ \dfrac{xT^*y}{xT^iy, xT^*y}$ $\quad (-^*)\ \dfrac{x-(T^*)y}{x-(T^0)y \mid \ldots \mid x-(T^i)y \mid \ldots}$

for any $i \geq 0$ where $T^0 = 1'$, $T^{i+1} = T\,;T^i$

Below we list the specific rules of $\mathsf{RL_{QV}}$-dual tableau.

For all object variables x, y, z and for every relational term $T \in \mathbb{RC}$:

$(1'1)\ \dfrac{xTy}{xTz, xTy \mid y1'z, xTy}$ $\quad (1'2)\ \dfrac{xTy}{x1'z, xTy \mid zTy, xTy}$

For every $\star \in \mathbb{C}$ and for all object variables x and y:

(right) $\dfrac{x\Psi_\star y}{x\Psi_\star z, x\Psi_\star y}$ for any object variable z

For every $T \in \{R_{(\mathsf{z},\mathsf{n})}\} \cup \{R_{(\mathsf{v_i},\mathsf{o_j})} \mid 1 \leq \mathsf{i} \leq 3, 1 \leq \mathsf{j} \leq 4\} \cup \{\Psi_\star \mid \star \in \mathbb{C}\}$ and for all object variables x and y:

(cut) $\dfrac{}{xTy \mid x-Ty}$

For all $\mathsf{v}, \mathsf{v_r}, \mathsf{v_s} \in L_1$, $\mathsf{o}, \mathsf{o_j}, \mathsf{o_{j+1}}, \mathsf{o_{j+2}} \in L_2$, and for all object variables x and y:

$(r1 \subseteq)\ \dfrac{xR_{(\mathsf{v},\mathsf{o})}y}{xR_{(\mathsf{v},\mathsf{o})}z, xR_{(\mathsf{v},\mathsf{o})}y \mid zR_{(\mathsf{z},\mathsf{n})}y, xR_{(\mathsf{v},\mathsf{o})}y}$ for any object variable z

$(r1 \supseteq)\ \dfrac{x-R_{(\mathsf{v},\mathsf{o})}y}{x-R_{(\mathsf{v},\mathsf{o})}z, z-R_{(\mathsf{z},\mathsf{n})}y, x-R_{(\mathsf{v},\mathsf{o})}y}$ for a new object variable z

$(r2 \subseteq)\ \dfrac{xR_{(\mathsf{z},\mathsf{n})}y}{xR_{(\mathsf{v},\mathsf{o_j})}z, xR_{(\mathsf{z},\mathsf{n})}y \mid zR_{(\mathsf{v},\mathsf{o_{j+2}})}y, xR_{(\mathsf{z},\mathsf{n})}y}$ for any object variable z and $\mathsf{j} \in \{1,2\}$

$(r2 \supseteq)\ \dfrac{x-R_{(\mathsf{z},\mathsf{n})}y}{x-R_{(\mathsf{v},\mathsf{o_j})}z, z-R_{(\mathsf{v},\mathsf{o_{j+2}})}y, x-R_{(\mathsf{z},\mathsf{n})}y}$ for a new object variable z and $\mathsf{j} \in \{1,2\}$

$(r3)\ \dfrac{x\Psi_{(\mathsf{v},\mathsf{o_j})}y, x\Psi_{(\mathsf{v},\mathsf{o_{j+1}})}y}{zR_{(\mathsf{v},\mathsf{o_{j+1}})}x, K \mid z\Psi_{(\mathsf{v},\mathsf{o_j})}y, K}$ for any object variable z, $\mathsf{j} \in \{1,2,3\}$

$K = x\Psi_{(\mathsf{v},\mathsf{o_j})}y, x\Psi_{(\mathsf{v},\mathsf{o_{j+1}})}y$

$(r4)\ \dfrac{x\Psi_{(\mathsf{v_s},\mathsf{o_{j+1}})}y}{zR_{(\mathsf{v_s},\mathsf{o_{j+1}})}x, x\Psi_{(\mathsf{v_s},\mathsf{o_{j+1}})}y \mid z\Psi_{(\mathsf{v_r},\mathsf{o_j})}y, x\Psi_{(\mathsf{v_s},\mathsf{o_{j+1}})}y}$ for any object variable z and $\mathsf{j} \in \{1,2,3\}$ and $\mathsf{r} < \mathsf{s}$

$$(r5)\quad \frac{x\Psi_{(\mathsf{v_s},\mathsf{o})}y,\, x\Psi_{(\mathsf{v_3},\mathsf{o})}y}{zR_{(\mathsf{v_s},\mathsf{o})}x, K \mid z\Psi_{(\mathsf{v_r},\mathsf{o})}y, K}$$

for any object variable z and $\mathsf{s} \in \{2,3\}$

$\mathsf{r} < \mathsf{s}$ and $K = x\Psi_{(\mathsf{v_s},\mathsf{o})}y, x\Psi_{(\mathsf{v_3},\mathsf{o})}y$

$$(r6)\quad \frac{x\Psi_{(\mathsf{v_s},\mathsf{o_{j+2}})}y,\, x\Psi_{(\mathsf{v_{s-1}},\mathsf{o_{j+2}})}y}{zR_{(\mathsf{v_s},\mathsf{o_{j+2}})}x, K \mid z\Psi_{(\mathsf{v_r},\mathsf{o_j})}y, K}$$

for any object variable z and $\mathsf{j} \in \{1,2\}$, $\mathsf{s} \in \{2,3\}$,

$\mathsf{r} < \mathsf{s}$, and $K = x\Psi_{(\mathsf{v_s},\mathsf{o_{j+2}})}y, x\Psi_{(\mathsf{v_{s-1}},\mathsf{o_{j+2}})}y$

A set of $\mathsf{RL_{QV}}$-formulas is said to be an $\mathsf{RL_{QV}}$-*axiomatic set* whenever it includes a subset of either of the following forms, for all object variables x, y, for every relational term T, for any $\star \in \mathbb{C}$, and for any $\# \in \mathbb{C} \setminus \{\star\}$:

(Ax1) $\{x1'x\}$

(Ax2) $\{x1y\}$

(Ax3) $\{xTy, x{-}Ty\}$

(Ax4) $\bigcup_{\star\in\mathbb{C}}\{x\Psi_\star y\}$

(Ax5) $\{x{-}\Psi_\star y, x{-}\Psi_\# y\}$

Let φ be an $\mathsf{RL_{QV}}$-formula. An $\mathsf{RL_{QV}}$-*proof tree for* φ is a tree with the following properties:

– The formula φ is at the root of this tree.
– Each node except the root is obtained by an application of an $\mathsf{RL_{QV}}$-rule to its predecessor node.
– A node does not have successors whenever its set of formulas is an $\mathsf{RL_{QV}}$-axiomatic set or none of the rules is applicable to its set of formulas.

Observe that the proof trees are constructed in the top-down manner, and hence every node has a single predecessor node.

A branch of an $\mathsf{RL_{QV}}$-proof tree is said to be *closed* whenever it contains a node with an $\mathsf{RL_{QV}}$-axiomatic set of formulas. A tree is *closed* iff all of its branches are closed. An $\mathsf{RL_{QV}}$-formula φ is $\mathsf{RL_{QV}}$-*provable* whenever there is a closed $\mathsf{RL_{QV}}$-proof tree for it which is then refereed to as its $\mathsf{RL_{QV}}$-*proof*.

Theorem 2 (Relational Soundness and Completeness).
For every QV*-formula* φ *and for all object variables* x *and* y*, the following conditions are equivalent:*

1. φ *is* QV*-valid.*
2. $x\tau(\varphi)y$ *is* $\mathsf{RL_{QV}}$*-provable.*

Example
Let φ be a QV-formula of the following form: $\varphi = (\mathsf{v},\mathsf{o_1}) \rightarrow [\otimes_{(\mathsf{v},\mathsf{o_2})}]((\mathsf{v},\mathsf{o_1}) \vee (\mathsf{v},\mathsf{o_2}))$.
The translation of φ into $\mathsf{RL_{QV}}$-term is: $\tau(\varphi) = -\Psi_{(\mathsf{v},\mathsf{o_1})} \cup -(R_{(\mathsf{v},\mathsf{o_2})} ; -(\Psi_{(\mathsf{v},\mathsf{o_1})} \cup \Psi_{(\mathsf{v},\mathsf{o_2})}))$
Figure 2 shows $\mathsf{RL_{QV}}$-proof of the formula $x\tau(\varphi)y$, which by Theorem 2 proves QV-validity of φ. In each node of the tree presented in the example we underline the formulas which determine the rule that has been applied during the construction of the tree and we indicate which rule has been applied. If a rule introduces a variable, then we write how the variable has been instantiated. Furthermore, in each node we write only those formulas which are essential for the application of a rule and the succession of these formulas in the node is usually motivated by the reasons of formatting.

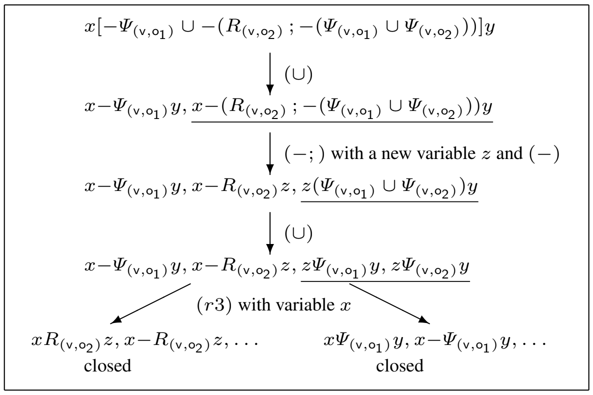

Fig. 2. $\mathsf{RL}_{\mathsf{QV}}$-proof of QV-validity of the formula $(\mathsf{v}, \mathsf{o}_1) \rightarrow [\otimes_{(\mathsf{v},\mathsf{o}_2)}]((\mathsf{v}, \mathsf{o}_1) \vee (\mathsf{v}, \mathsf{o}_2))$.

5 Conclusions and future work

We have presented a sound and complete relational dual tableau for verification of validity of QV-formulas. This system is a first step in order to provide a general framework for improving the capacity of reasoning about moving objects. The direction of our future work is twofold. First of all, we will focus on the extension of the logic by considering other spatial components (relative position, closeness, etc.). On the other hand, it would be needed a prover which is a decision procedure based on the dual tableau presented in this paper.

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