# MAT Logic: A Temporal×Modal Logic with Non-Deterministic Operators to Deal with Interactive Systems in Communication Technologies

Aguilera, G., Burrieza*, A., Cordero, P., P. de Guzmán, I. and Muñoz, E.

Departamento de Matemática Aplicada, * Departamento de Filosofía.
Universidad de Málaga. 29071-Spain
gabri@ctima.uma.es, burrieza@uma.es, pcordero@uma.es,
guzman@ctima.uma.es, emilio@ctima.uma.es

**Abstract.** In this paper, the Multi-flow Asynchronous Temporal Logic, called MAT Logic, is presented. MAT Logic is a new temporal×modal logic with non-deterministic operators among time flows as accessibility relations. The main goal of this work has been the design and description of a logic that could be capable of managing communications among systems with not necessarily synchronizable time flows. In order to better understand the design of the logic, an example in the field of communications is given.

## 1 Introduction

The necessity of the incorporation of non-determinism in computation has been widely discussed. So, for example, in the literature, the concept of non-deterministic automata as a formal model of computation is widely consolidated; in [20] the author presents a discussion about how the study of non-determinism is useful for natural language processing; in [10] the author shows how formal non-deterministic models are useful in describing interactive systems. Another example is designing a circuit or a network: non-determinism characterizes the flexibility allowed in the design [15].

Most works about non-determinism are based on simulation by means of algorithms and deterministic automata. Nonetheless, it is widely accepted that it will be necessary to develop a formal theory that regards non-determinism as inherent to it and the fact that computational logic will play an important role in this development [12].

Thus, on the one hand, modal logics have been proven useful in interactive systems. So, they have been used in multi-agent systems to describe the agent mental state and behaviour [17], or, for example, to reason with social categories, such as obligations [3] and cooperativity [1].

On the other hand, temporal logic has been shown as a successful tool for specifying and reasoning with interactive systems and the global behaviour of multi-agent systems. However, it is not capable of reasoning out the intern structure of these systems [8, 13, 16]. In the literature, there exist several extensions of propositional temporal logic to solve this disadvantage. So, for example, in the case of multi-agent systems, the simplest extension is to consider that all the agents are synchronized [9, 16], nevertheless, this is a very strong restriction. Other extensions are obtained via some form of synchronization given by *visibility* or *accessibility* functions. Thus, temporal logics with linear temporal flows in which the visibility functions are bidirectional, that is, the relation among states (in different flows) is *symmetric*, were introduced in [14, 18].

In our opinion, a combination of the above approaches, i.e. modal and temporal logics, could be the key to achieve a more comprehensive way to describe interactive and multi-agent systems. Nonetheless, determining which properties of the chosen combinations hold is not an easy task [21]. In the framework of combining this kind of logics, this work presents the Multi-flow Asynchronous Temporal Logic (briefly, MAT Logic). Our main goal has been the design and description of a logic that could be capable of managing communications among systems with time flows not necessarily synchronized. Occasionally, this kind of communication between two time flows can be described by a function. However, on many occasions the type of instant (kind of state of the system) in the image flow is known but not the specific instant, consequently, a function can not be defined. These characteristics, together with the fact that synchronization of the time flows is not required, have led us to represent accessibility among them by means of, on the one hand, *non-deterministic operators* for possible communications and, on the other hand, *execution functions* for effective communications.

The usual way in the literature about temporal×modal logics is to use equivalence relations of accessibility, for example the Kamp-models in [19] and the reasoning about knowledge and time in asynchronous systems, in [11]. However, in [6, 7] a new kind of frame was introduced to manage linear time flows connected by accessibility functions instead of using equivalence relations. MAT Logic is a more general framework, because the accessibility is given by non deterministic operators and, as a consequence, can be applied to different situations only by changing the properties required to them. Another characteristic to be considered, is the use of indexed connectives to label time flows that can be reached from the current flow (as in [7]). This notation, claimed by applications to interactive systems, allows us to identify systems which we want to establish a communication with.

Before the description of our logic, in order to better understand the aim in its design, we give the following simplified example. Consider a computer network with some physical links among them and, for simplifying, let us suppose that the possible states of the computers are: ready to send and receive, ready to send but not to receive, not ready to send but ready to receive and, finally, not ready to send and not ready to receive. Assume also that the computers are working changing their states and, in each change of state, a change in the instant in its time flow is produced. That is, the time flow of each computer represents the different states of this computer with respect to the time course. For simplicity in this example, we reason only with two computers, but these ideas can be easily generalizable for more computers. If a computer $X_1$ is planning to establish a communication with another $X_2$ in an instant $t_1 \in T_1$, being $T_i$ the time flow of $X_i$ for $i \in \{1, 2\}$, $t_1$ has to be a *ready to send* state and $X_2$ has to be in a *ready to receive* state. However, the specific instant in which the communication is executed is not initially known. As a consequence, the possible communications are represented by a subset of $T_2$, which is the image of $t_1$ by the accessibility non deterministic operator. Moreover, if the communication from $t_1$ is effectively executed in the instant $t_2 \in T_2$, then $t_2$ is the image of $t_1$ by the execution function and we assume that the images of every later instant of $t_1$ are lower bounded by $t_2$, because in this moment the information in $X_1$ about $X_2$ has been updated. The following figures represent two different situations.

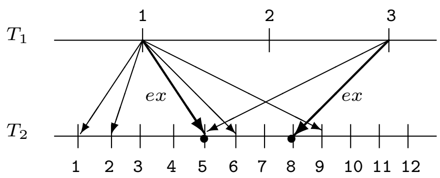



*figure 1*

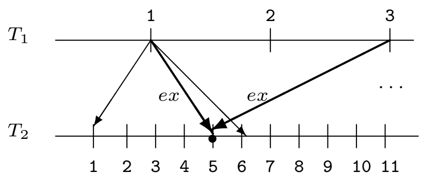



*figure 2*

In figure 1, the image of instant 1 of $T_1$ are 1, 2, 5 and 9 in $T_2$, which are possible instants in ready to receive state. Also, in instant 5 of $T_2$ a real communication (an execution) occurs. Instant 3 of $T_1$ is analogous. The image of 3 is lower bounded by the execution instant of 1, that is, 5 of $T_2$.

In figure 2, two different executions from instants 1 and 3 of $T_1$ occur in the same instant 5 of $T_2$. This can be explained because the instant 5 of $T_2$ can be in a *ready to receive* state for which communication with instant 1 of $T_1$ occurred, but due to the computer $X_2$ do not change its state, a communication with 3 of $T_1$ occurred also.

The figures above give the intuitive idea about the type of frame that we are going to define in this paper: different time flows and the accessibility between every pair of them is given by a lower bounded non-deterministic operator (possible communications) and an execution function (effective communications) which determines these lower bounds. This kind of frame, as we will see, will allow us to interpret temporal×modal connectives of our MAT logic.

This paper is organized as follows: In section 2, concepts of non-deterministic operator and lower bounded non-deterministic operator are introduced. Notation that will be used in the rest of the paper is introduced too. In section 3, MAT logic is defined. Moreover, the semantic is shown, emphasizing the set of *accessibility non deterministic operators* among temporal flows, $\mathcal{C}$, the set of *execution functions*, $\mathcal{F}_{ex}$. In section 4, an axiom system $S_{MAT}$ for our logic is introduced. Also, soundness and completeness of the system are stated. Finally, in section 5, some conclusions and future works are shown.

## 2 Lower Bounded Unary Non-Deterministic Operators

This section is devoted to the necessary preliminaries about non-deterministic operators.

**Definition 1.** *Let $A$ and $B$ be non-empty sets and $n \in \mathbb{N}$ where $n \geq 1$. Any function $F : A^n \to 2^B$ is said to be a* **non-deterministic operator of arity** $n$ *from $A$ to $B$. Any non-deterministic operator of arity 1 from $A$ to $B$ is called a* **ndo** *from $A$ to $B$. The set $\mathcal{N}do(A, B)$ is the set of all non-deterministic operators of arity 1 from $A$ to $B$.*

In the same way that occurs when we work with functions, $F(X)$ will denote the set $\bigcup_{x\in X} F(x)$, for all $F \in \mathcal{N}do(A, B)$ and $X \subseteq A$.

**Definition 2.** *Given two non-empty sets $A$ and $B$, the relation $\subseteq$ in $\mathcal{N}do(A, B)$ is defined by $F \subseteq G$ if and only if $F(a) \subseteq G(a)$ for all $a \in A$.*

*Remark 1.* Non-determinism condition is about the fact that cardinality of the images is arbitrary, contrarily to functions and deterministic operators. Nevertheless, every (total or partial) function $f : A \to B$ can be identified with an element of $F \in \mathcal{N}do(A, B)$:

$$F : A \to 2^B \qquad \text{and} \qquad F(a) = \begin{cases} \{f(a)\}, & \text{if } f \text{ is defined for } a; \\ \varnothing, & \text{if } f \text{ is not defined for } a. \end{cases}$$

In this work, functions will be considered in this previous way. This fact motivates the following definition.

**Definition 3.** *Let $A$ and $B$ be non-empty sets and $F \in \mathcal{N}do(A, B)$, we define the* **domain** *of $F$ as the set $Dom(F) = \{a \in A \mid F(a) \neq \varnothing\}$. The* **empty ndo***, denoted by $\emptyset$, is the ndo whose domain is empty, that is, $\emptyset : A \to 2^B$ and $\emptyset(a) = \varnothing$, for all $a \in A$.*

As it was mentioned in the introduction, we are interested in linear temporal flows and particularly in the use of ndos with the characteristics collected in the following definition.

**Definition 4.** *Let $A$ and $B$ be two linear ordered sets and let $F$ be a ndo from $A$ to $B$. $F$ is* **lower bounded** *if, for all $a \in Dom(F)$, the minimum of $F(a)$ exists (hereinafter denoted $\min F(a)$). $\mathcal{N}do^{lb}(A, B)$ denotes the set of all lower bounded ndos of arity 1 from $A$ to $B$, its elements will be called lb-ndo.*

Some notations useful in the rest of the paper are introduced now.

**Notation:** Let $(A, \leq)$ be a linear ordered set, $a$ be an element of $A$ and $X \subseteq A$.
$[a, \to) = \{x \in A \mid a \leq x\}$, $X\!\uparrow = \bigcup_{x\in X} [x, \to)$ and $X\!\uparrow^* = \bigcup_{x\in X} (x, \to)$.
$(a, \to)$, $(\leftarrow, a]$, $(\leftarrow, a)$, $X\!\downarrow$ and $X\!\downarrow^*$ can be analogously defined.

## 3 The MAT Logic

In this section MAT logic is defined as a family of indexed temporal×modal logics $MAT\text{-}\mathfrak{I} = (L_\mathfrak{I}, \mathcal{M}^\mathfrak{I})$ where $\mathfrak{I}$ is a non-empty numerable set of indexes. The selection of this set determines a specific MAT logic. $L_\mathfrak{I}$ denotes the language and $\mathcal{M}^\mathfrak{I}$ the set of models for $L_\mathfrak{I}$.

### 3.1 The Language $L_\mathfrak{I}$ of $MAT\text{-}\mathfrak{I}$

Given a denumerable set of indexes $\mathfrak{I}$, the alphabet of $L_\mathfrak{I}$ consists of:

(i) a denumerable set, $\mathcal{V}$, of propositional variables;
(ii) the logic constants $\top$ ("truth") and $\bot$ ("falseness"), and the boolean connectives $\neg$ ("not"), $\wedge$ ("and"), $\vee$ ("or") and $\rightarrow$ ("if... then..." );
(iii) the temporal connective of future $G$ ("it will always be that") and $H$ ("it has been always that");
(iv) the three indexed modal connectives $<i>$, $<i>_{min}$ and $<i>_{ex}$ for $i \in \mathfrak{I}$;
(vi) the auxiliary symbols: (, ).

The well formed formulae (*wffs*) are generated by the construction rules of classical propositional logic by adding the new rule:
*If $A$ is a* wff*, then $GA$, $HA$. $<i>A$, $<i>_{min}A$ and $<i>_{ex}A$ are* wffs. The desired interpretation of the new modal connectives is as follows:

- $<i>A$ is read as "*There exists a temporal flow $T_i$ and there exist some states in $T_i$ that are available from present state and $A$ is true in some of these states*".
- $<i>_{min}A$ is read as "*There exists a temporal flow $T_i$ and there exist some states in $T_i$ that are available from present state and $A$ is true in the minimum of these states*".
- $<i>_{ex}A$ is read as "*There exists a temporal flow $T_i$ and there exist some states in $T_i$ that are available from present state and $A$ is true in one of these states, specifically in the execution state*".

We also consider the connectives $[\,i\,]$, $[\,i\,]_{min}$ and $[\,i\,]_{ex}$ as usual in modal logic.

### 3.2 Semantics of $MAT\text{-}\mathfrak{I}$

As we have said in the introduction section, the frames must satisfy some properties formalized in the following definition.

**Definition 5.** *A MAT-* **frame** *is a tuple $\Sigma = (W, \Lambda, \mathcal{T}, \mathcal{C}, \mathcal{F}_{ex})$ such that:*

*(1) $W$ is a non-empty set (set of labels that will be used for temporal flows).*
*(2) $\Lambda$ is a distinguished subset (possibly empty) of $W$.*
*(3) $\mathcal{T} = \{(T_w, <_w) \mid w \in W\}$ is a non-empty set of temporal flows, such that $T_w \neq \varnothing$ for all $w \in W$; $T_w \cap T_{w'} = \varnothing$ for all $w, w' \in W$ with $w \neq w'$ and, for all $w \in W$, $<_w$ is a strict order relation in $T_w$ which is linear. The elements $t_w$ of the disjoint union $Coord_\Sigma = \bigoplus_{w \in W} T_w$ are called* **coordinates***.*

(4) $\mathcal{C}$ *is a set of ndos* $\mathcal{C} = \{C_w^l \mid (w,l) \in W \times \Lambda\}$ *whose elements, called* **accessibility ndos**, *satisfy that for any* $(w,l) \in W \times \Lambda$, $C_w^l$ *is an lb-ndo (possibly the empty ndo,* $\emptyset$*) from* $T_w$ *to* $T_l$.

(5) $\mathcal{F}_{ex}$ *is a set of partial functions* $\mathcal{F}_{ex} = \{\xrightarrow{w\ l}_{ex} \mid (w,l) \in W \times \Lambda\}$, *whose elements, called* **execution functions**, *satisfy that for any* $(w,l) \in W \times \Lambda$, $\xrightarrow{w\ l}_{ex}$ *is a partial function (possibly the empty function,* $\emptyset$*) from* $T_w$ *to* $T_l$.

(6) $\mathcal{C}$ *and* $\mathcal{F}_{ex}$ *satisfy the following conditions:*

6.1) $\xrightarrow{w\ l}_{ex} \subseteq C_w^l$ *for all* $(w,l) \in W \times \Lambda$

6.2) *If* $t_w \in Dom(\xrightarrow{w\ l}_{ex})$, *then* $C_w^l\Big((t_w, \rightarrow)\Big) \subseteq \big(\xrightarrow{w\ l}_{ex}(t_w)\big)\uparrow$

*Remark 2.* Condition (6) gives the relation between the accessibility ndos and execution functions: (6.1) says that the image of the execution function is a subset of the image of the corresponding ndo, that is, the execution instant in one of the available states and (6.2) represents that the execution instant is a lower bound for the image by the ndo of later states, as we have said previously.

The following definition introduces a lb-ndo useful in the rest of the paper.

**Definition 6.** *Given a MAT-frame* $(W, \Lambda, \mathcal{T}, \mathcal{C}, \mathcal{F}_{ex})$ *and* $(w,l) \in W \times \Lambda$, *we define the lb-ndo,* $m_l : T_w \to 2^{T_l}$, *as follows:*

$$m_l(t_w) = \begin{cases} \{\min C_w^l(t_w)\}, & \text{if } t_w \in Dom(C_w^l) \\ \varnothing, & \text{otherwise} \end{cases}$$

Now, we have all the necessary elements to define semantics of the connectives of $L_{\mathfrak{I}}$. $\mathfrak{I}$ will play a notable role in this semantic (as an arsenal of names for denoting the range of lb-ndos).

**Definition 7.** *A* **model** *for* $L_{\mathfrak{I}}$ *is an ordered pair* $\mathcal{M}^{\mathfrak{I}} = (\Sigma^{\mathfrak{I}}, h)$, *where:*

i) $\Sigma^{\mathfrak{I}}$ *is a MAT-frame,* $\Sigma^{\mathfrak{I}} = (W, \Lambda^{\mathfrak{I}}, \mathcal{T}, \mathcal{C}, \mathcal{F}_{ex})$, *such that* $\Lambda^{\mathfrak{I}} = W \cap \mathfrak{I}$. *From now on,* $\Sigma^{\mathfrak{I}}$ *will be called a* **MAT-frame depending on** $\mathfrak{I}$.

ii) *A function* $h : \mathcal{V} \longrightarrow 2^{Coord_{\Sigma^{\mathfrak{I}}}}$, *assigning each atom* $p \in \mathcal{V}$ *a subset of* $Coord_{\Sigma^{\mathfrak{I}}}$ *is called an* **interpretation**.

*The interpretation h is recursively extended to a function (still denoted h) defined on all formulae of* $L_{\mathfrak{I}}$ *that satisfies usual conditions for boolean and temporal connectives. Moreover, for our special modal connectives, we have:*

$$h(<i>A) = \begin{cases} \{t_w \in Coord_{\Sigma^{\mathfrak{I}}} \mid C_w^i(t_w) \cap h(A) \neq \varnothing\} & \text{if } i \in \Lambda^{\mathfrak{I}}, \\ \varnothing & \text{otherwise;} \end{cases}$$

$$h(<i>_{min}A) = \begin{cases} \{t_w \in Coord_{\Sigma^{\mathfrak{I}}} \mid m_i(t_w) \cap h(A) \neq \varnothing\} & \text{if } i \in \Lambda^{\mathfrak{I}}, \\ \varnothing & \text{otherwise;} \end{cases}$$

$$h(<i>_{ex}A) = \begin{cases} \{t_w \in Coord_{\Sigma^{\mathfrak{I}}} \mid \xrightarrow{w\ i}_{ex}(t_w) \cap h(A) \neq \varnothing\} & \text{if } i \in \Lambda^{\mathfrak{I}}, \\ \varnothing & \text{otherwise;} \end{cases}$$

*Remark 3.* As expressed in Remark 1, $\overset{w\ i}{\longrightarrow}_{ex}(t_w)$ is considered as a subset of $T_i$.

Now, we have the necessary elements for the formal definition of the concepts validity, truth and satisfiability.

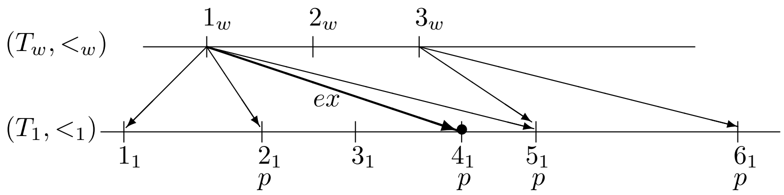
**Definition 8.** *We say that a formula $A$ of $L_{\mathfrak{I}}$ is* **satisfiable** *if there exists a model $\mathcal{M}^{\mathfrak{I}} = (\Sigma^{\mathfrak{I}}, h)$ for $L_{\mathfrak{I}}$, and $t_w \in \mathcal{C}oord_{\Sigma^{\mathfrak{I}}}$ such that $t_w \in h(A)$; in this case we also say that $A$ is* **true at** $t_w$. *$A$ is said to be* **valid in a model** $\mathcal{M}^{\mathfrak{I}} = (\Sigma^{\mathfrak{I}}, h)$ *for $L_{\mathfrak{I}}$, if $A$ is true at every coordinate in the model, that is, if $h(A) = \mathcal{C}oord_{\Sigma^{\mathfrak{I}}}$. Let $\Sigma^{\mathfrak{I}}$ be a MAT-frame depending on $\mathfrak{I}$. We say that $A$ is* **valid in** $\Sigma^{\mathfrak{I}}$, *if $A$ is valid in every model $\mathcal{M}^{\mathfrak{I}} = (\Sigma^{\mathfrak{I}}, h)$. Let $\mathbb{K}$ be a class of MAT-frames depending on $\mathfrak{I}$. We say that $A$ is* **valid in the class** $\mathbb{K}$ *if $A$ is valid in all MAT-frames $\Sigma^{\mathfrak{I}} \in \mathbb{K}$. If $\mathbb{K}$ is the class of all MAT-frames depending on $\mathfrak{I}$, then we simply say that $A$ is* **valid**.

The following example allows us to comment the semantics of the specific modal connectives. Given a language $L_{\mathfrak{I}}$ being $\mathfrak{I} = \{0, 1\}$, we define the MAT-frame depending on $\mathfrak{I}$, $\Sigma^{\mathfrak{I}} = (W, \Lambda^{\mathfrak{I}}, \mathcal{T}, \mathcal{C}, \mathcal{F}_{ex})$ such that:

- $W = \{w, 1\}$, thus by definition we have that $\Lambda^{\mathfrak{I}} = \{1\}$.
- $\mathcal{T} = \{(T_w, <_w), (T_1, <_1)\}$, where $T_w = \{1_w, 2_w, 3_w\}$, $T_1 = \{1_1, 2_1, 3_1, 4_1, 5_1, 6_1\}$ and $<_w, <_1$ the usual strict linear order relations in $T_w$ and $T_1$, respectively.
- $\mathcal{C} = \{C_1^1, C_w^1\}$, being $C_1^1 = \emptyset_1^1$, $C_w^1(1_w) = \{1_1, 2_1, 4_1, 5_1\}$, $C_w^1(2_w) = \varnothing$ and $C_w^1(3_w) = \{5_1, 6_1\}$
- $\mathcal{F}_{ex} = \{\overset{1\ 1}{\longrightarrow}_{ex}, \overset{w\ 1}{\longrightarrow}_{ex}\}$, where $\overset{1\ 1}{\longrightarrow}_{ex} = \emptyset_1^1$ and $\overset{w\ 1}{\longrightarrow}_{ex}$ is defined by: $\overset{w\ 1}{\longrightarrow}_{ex}(1_w) = \{4_1\}$; $\overset{w\ 1}{\longrightarrow}_{ex}(2_w) = \varnothing$; $\overset{w\ 1}{\longrightarrow}_{ex}(3_w) = \varnothing$.

Consequently we have $m_1(1_w) = \{1_1\}$; $m_1(2_w) = \varnothing$; $m_1(3_w) = \{5_1\}$ and, for all $t_1 \in T_1$, $m_1(t_1) = \varnothing$.

If $p$ is a formula, we can define a model $(\Sigma^{\mathfrak{I}}, h)$ such that $h(p) = \{2_1, 4_1, 5_1, 6_1\}$. The following figure represents this model.




In this model, statements about truth or falseness of some formulae in coordinates of $T_w$ are collected in the following table:

| | |
|---|---|
| $<1>\, p$ is true in $1_w$ and is true in $3_w$ | $<1>\, p$ is false in $2_w$ |
| $<1>_{min}\, p$ is true in $3_w$ | $<1>_{min}\, p$ is false in $1_w$ and $2_w$ |
| $<1>_{ex}\, p$ is true in $1_w$ | $<1>_{ex}\, p$ is false in $2_w$ and $3_w$ |
| $[\,1\,]\, p$ is true in $2_w$, $3_w$ and in all $t_1 \in T_1$ | $[\,1\,]\, p$ is false in $1_w$ |
| $[\,1\,]_{min}\, p$ is true in $2_w$, $3_w$ and in all $t_1 \in T_1$ | $[\,1\,]_{min}\, p$ is false in $1_w$ |
| $[\,1\,]_{ex}\, p$ is true in all $t_w \in T_w$ and in all $t_1 \in T_1$ | |

In the context of computer network, if we interpret the formula $p$ by *the computer is running a specific program* $P$ then, for example, in $1_w$ the formula $F[\,1\,]\,p$ is true and $H <1>_{ex}\, p$ is true in $2_w$. The first formula means that in a future state (in this case $3_w$), all the available states in $T_1$ ($5_1$ and $6_1$) will be running the program $P$. The second formula means that always in the past of $2_w$ (that is, in $1_w$) the execution state in $T_1$ (in this case, $4_1$) was running the program $P$.

## 4 The Axiom System $\mathcal{S}_{MAT}$

In this section an axiom system for the language $L_{\Im}$ is introduced denoted by $\mathcal{S}_{MAT}$. This system has the following schema of axioms and inference rules.

1. Propositional linear temporal logic schema $\mathcal{K}_l$
2. For each $i \in \Im$ standard modal schemas of axioms:
   2.1 $[\,i\,](A \to B) \to ([\,i\,]A \to [\,i\,]B)$.
   2.2 $[\,i\,]_{min}(A \to B) \to ([\,i\,]_{min}A \to [\,i\,]_{min}B)$.
   2.3 $[\,i\,]_{ex}(A \to B) \to ([\,i\,]_{ex}A \to [\,i\,]_{ex}B)$.
3. For each $i \in \Im$, specific schemas of axioms:
   3.1 $<i>A \to <i>_{min}(A \vee FA)$
   3.2 $<i>_{min}A \to [\,i\,]_{min}A$
   3.3 $<i>_{ex}(A \wedge GA) \to G[\,i\,]A$
   3.4 $<i>_{ex}A \to [\,i\,]_{ex}A$
   3.5 $(\lambda\, A \,\wedge \lambda'\, B) \to \lambda\,(A \wedge (PB \vee B \vee FB))$, where:

$$\begin{cases} (\dagger)_1 \quad \lambda = \gamma_1 <j_1>_{min}\gamma_2 \dots <j_n>_{min} \quad \text{with} \begin{cases} \gamma_l \in \{P, F, \epsilon\}, \\ j_l \in \Im, \text{ and} \\ 1 \le l \le n. \end{cases} \\ (\dagger)_2 \quad \lambda' = \gamma'_1 <k_1>_{min}\gamma'_2 \dots <k_s>_{min} \quad \text{with} \begin{cases} \gamma'_l \in \{P, F, \epsilon\}, \\ k_l \in \Im, \text{ and} \\ 1 \le l \le s. \end{cases} \\ (\dagger)_3 \quad j_n = k_s \end{cases}$$

   3.6 $<i>_{min}A \to <i>A$
   3.7 $<i>_{ex}A \to <i>A$

The inference rules are propositional linear temporal logic $\mathcal{K}_l$ inference rules together with the rule: For all $i \in \Im$: $\dfrac{A}{[\,i\,]A}$

Informal reading of specific modal axioms is the following:

3.1 and 3.2 formalize existence and uniqueness of the minimum available state, respectively.
3.3 ensures that available states from a given one are lower bounded by execution states.
3.4 means that if there exists the execution instant then it is unique.

3.5 say that every two access chains to the same time flow (because $j_n = k_s$) through the minima converge.
3.6 and 3.7 ensure that minimum and execution states are available states, respectively.

Syntactical concepts as *Proof* or *Theorem* are defined as usual.

**Theorem 1.** *System $S_{MAT}$ is sound and complete.*

The soundness of $S_{MAT}$ can be obtained by proving the validity of the axioms and taking into account that the inference rules are validity-preserving.

Regarding completeness, a *step-by-step* proof (see, for example, [2, 4, 6, 7]) can be given in the following terms: Given any consistent formula $A$ , we have to prove that $A$ is satisfiable. With this purpose, the step-by-step method defines a MAT-frame $\Sigma$ and a function $\Phi_\Sigma$ which assigns maximal consistent sets to any coordinate, such that $A \in \Phi_\Sigma(t_w)$ for some $t_w \in \mathcal{C}oord_\Sigma$. The process to build such a frame is recursive, successive extensions of the frames are defined until $\Sigma$ is obtained.

Due to lack of space, the formal details of soundness and completeness proofs are left for a longer version of this paper.

## 5 Conclusions and Future Work

A new combination of modal and temporal logic, called MAT logic has been presented. This logic allows us to manage communications among systems without synchronization restrictions. The achievement of this goal has been possible due to the use of non-deterministic operators among time flows. Together with the semantic of the MAT logic, defined in an algebraic style, a sound and complete axiom system $S_{MAT}$ has been introduced.

At the present time, we are looking for formulae whose validity characterizes important properties for the communications of systems to extend the field of application of our MAT logic, for example in the planning area.

We are also studying the possibility of increasing the expressivity of MAT-Logic combining a totally expressive temporal logic (concretely, LN Logic [5]) with a modal logic in the same way used in this paper.

Last bust not least, it is planned to design a method for automated deduction in MAT-Logic.

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