On the modal logic of order-of-magnitude qualitative reasoning: a tableau calculus

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***Abstract*— This work is based on the multimodal logic $\mathcal{L}(MQ)$, recently introduced, which formalizes order-of-magnitude qualitative reasoning. The aim of this paper is to provide a sound and complete tableau method for the future fragment of $\mathcal{L}(MQ)$.**

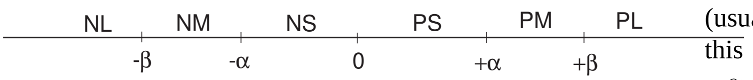
I. Introduction

Several reasoning methods have been devised in order to face a problem one often encounters in real world applications, which is a lack of quantitative (numerical) information among the observed facts. Among these methods we find those which allow for reasoning under an incompletely specified environment, giving rise to reasoning schemes for fuzzy, imprecise and missing information. A different approach is to apply ideas from qualitative reasoning and, specifically, from order-of-magnitude reasoning (OMR), which was introduced in [10] and later extended in [6], [7], [9], [14], [15].

The basis of OMR systems is computing with a set of coarse values, usually generated as abstract representations of precise values. This is of course the same approach taken by any qualitative reasoning system. The distinctive feature of OMR is that the coarse values are generally of different order of magnitude.

Depending on the way the coarse values are defined, different OMR calculi can be generated: It is usual to distinguish between Absolute Order of Magnitude (AOM) and Relative Order of Magnitude (ROM) models.

There exist attempts to integrate both approaches, so that an absolute partition is combined with a set of comparison relations between real numbers [14], [15]. For instance, it is usual to consider the $AOM(5)$ approach which, by considering five landmarks, it is customary to divide the real line in seven equivalence classes and use the following labels to denote these equivalence classes of $\mathbb{R}$:



The labels correspond to "negative large", "negative medium", "negative small", "zero", "positive small", "positive medium" and "positive large", respectively. The real numbers α and β are the landmarks used to delimit the equivalence classes (the particular criteria to choose these numbers would depend on the application in mind). In [14] three binary relations (*close to, comparable, negligible*) were defined in the spirit of [10], but using the labels corresponding to quantitative values, and preserving coherence between the relative model they define and the absolute model in which they are defined.

Although the use of qualitative OMR has been an active research area in AI for some time, the analogous development of a logical approach has received little attention. Various multimodal approaches have been promulgated, for example, for qualitative spatial and temporal reasoning but, as far as we know, no such approach has been developed for OMR. However, non-classical logics do have been used as a support of qualitative reasoning in several ways: among the formalisms for qualitative spatial reasoning, the Region Connection Calculus (RCC) [11], [1] has received particular attention; in [2], [16], multimodal logics were used to deal with qualitative spatio-temporal representations, and in [13] branching temporal logics have been used to describe the possible solutions of ordinary differential equations when we have limited information about a system.

Recently, in [4], the logic $\mathcal{L}(MQ)$ for qualitative order-of-magnitude reasoning was introduced to handle, in some sense, the notion of comparability. Then, the same authors extended the logic to introduce also modalities for the negligibility relation [5] but without considering any attempt to mechanise its deduction. The purpose of this paper is to develop a non-classical logic for handling qualitative reasoning with orders of magnitude.

As a first approach to the logics of qualitative order-of-magnitude reasoning, we have based our minimal languages on the system $AOM(2)$, which is both simple enough to keep under control the complexity of the system and rich enough so as to permit the representation of a subset of the usual language of qualitative order-of-magnitude reasoning.

The intuitive representation of the underlying set of values (usually considered to be subsets of the real numbers, although this is not essential) is given below, in which two landmarks $-\alpha$ and $+\alpha$ are considered

In the picture, $-\alpha$ and $+\alpha$ represent respectively the greatest negative observable and the least positive observable. This choice makes sense, in particular, when considering physical metric spaces in which we always have a smallest unit which can be measured; however, it is not possible to identify a least or greatest non-observable number.

Once we have the equivalence classes in the real line, we can make comparisons between numbers by using binary relations such as

- x *is less than* y, in symbols $x < y$
- x *is less than and comparable to* y, in symbols $x \sqsubset y$.

where $\sqsubset$ is a restriction of the usual order of the real numbers ($<$) to numbers belonging to the same equivalence class.

Our aim in this paper is to provide tableau system for the future fragment of $\mathcal{L}(MQ)$. It is worth to remark that the need of considering the landmarks $-\alpha$ and $+\alpha$ as part of the frames makes more difficult the proof of completeness in $\mathcal{L}(MQ)$.

The rest of the paper is organized as follows: In Section II the syntax and the semantics of the proposed logic is introduced; in Section III, a tableau system is presented for the future fragment of $\mathcal{L}(MQ)$; then, in Section IV the completeness proof for the tableau system is given. Finally, some conclusions are drawn and prospects for future work are presented.

II. Syntax and Semantics of the Language $\mathcal{L}(MQ)$

In our syntax we will consider the connectives $\overrightarrow{\Box}$ and $\overleftarrow{\Box}$ to deal with the usual ordering $<$, the connectives $\overrightarrow{\blacksquare}$ and $\overleftarrow{\blacksquare}$ to deal with $\sqsubset$. The intuitive meanings of each modal connective is as follows:

- $\overrightarrow{\Box}A$ means A *is true for all number greater than the current one*.
- $\overrightarrow{\blacksquare}A$ is read A *is true for all point greater than and comparable to the current one.*
- $\overleftarrow{\Box}A$ means A *is true for all number less than the current one*.
- $\overleftarrow{\blacksquare}A$ means A *is true for all number less than and comparable to the current one*.

The alphabet of the language $\mathcal{L}(MQ)$ is defined by using:

- A stock of atoms or propositional variables, $\mathcal{V}$.
- The classical connectives $\neg, \wedge, \vee$ and $\rightarrow$ and the constants $\top$ and $\bot$.
- The unary modal connectives and $\overrightarrow{\Box}$ and $\overleftarrow{\Box}$, $\overrightarrow{\blacksquare}$ and $\overleftarrow{\blacksquare}$.
- The constants α^+ and α^-
- The auxiliary symbols: (,).

Formulas are generated from $\mathcal{V} \cup \{\alpha^+, \alpha^-, \top, \bot\}$ by the construction rules of classical propositional logic adding the following rule: If A is a formula, then so are $\overrightarrow{\Box}A$, $\overleftarrow{\Box}A$, $\overrightarrow{\blacksquare}A$ and $\overleftarrow{\blacksquare}A$.

Definition 1: A *multimodal qualitative frame* for $\mathcal{L}(MQ)$ (or, simply, a *frame*) is a tuple $\Sigma = (\mathbb{S}, +\alpha, -\alpha, <)$, where

1) $\mathbb{S}$ is a nonempty set.[1].
2) $<$ is a strict linear order on $\mathbb{S}$.
3) $+\alpha$ and $-\alpha$ are designated points in $\mathbb{S}$ (called *frame constants*), and allow to form the sets OBS^+, INF, and OBS^- defined below:

$$OBS^- = \{x \in \mathbb{S} \mid x \leq -\alpha\} \qquad INF = \{x \in \mathbb{S} \mid -$$
$$OBS^+ = \{x \in \mathbb{S} \mid +\alpha \leq x\}$$

We will use $x \sqsubset y$ as an abbreviation of "$x < y$ and $x, y \in EQ$, where $EQ \in \{OBS^+, INF, OBS^-\}$".

Definition 2: Let Σ be a multimodal qualitative frame, a *multimodal qualitative model on* Σ (or Σ*-model*, for short) is an ordered pair $\mathcal{M} = (\Sigma, h)$, where h is a *meaning function* (or, *interpretation*) $h\colon \mathcal{V} \longrightarrow 2^{\mathbb{S}}$. Any interpretation can be uniquely extended to the set of all formulas in $\mathcal{L}(MQ)$ (also denoted by h) by using the usual conditions for the classical boolean connectives and the constants $\top$ and $\bot$, and the following conditions for the modal operators and frame constants:

$$\begin{aligned}
h(\overrightarrow{\Box}A) &= \{x \in \mathbb{S} \mid y \in h(A) \text{ for all } y \text{ such that } x < y\}\\
h(\overrightarrow{\blacksquare}A) &= \{x \in \mathbb{S} \mid y \in h(A) \text{ for all } y \text{ such that } x \sqsubset y\}\\
h(\overleftarrow{\Box}A) &= \{x \in \mathbb{S} \mid y \in h(A) \text{ for all } y \text{ such that } y < x\}\\
h(\overleftarrow{\blacksquare}A) &= \{x \in \mathbb{S} \mid y \in h(A) \text{ for all } y \text{ such that } y \sqsubset x\}\\
h(\alpha^+) &= \{+\alpha\} \qquad h(\alpha^-) = \{-\alpha\}
\end{aligned}$$

The concepts of truth and validity are defined in a straightforward manner.

III. A tableau system for the future fragment of $\mathcal{L}(MQ)$

In this section we develop a tableau system for the future fragment of the language $\mathcal{L}(MQ)$, denoted $\mathcal{L}(MQ+)$. Although originally inspired on the method presented by Goré [8], a number of non-trivial modifications have been included due to the particularities of $\mathcal{L}(MQ+)$. Firstly, note that it is possible to semantically prove the following equivalence:

$$\overrightarrow{\blacksquare}A \leftrightarrow \Big(\alpha^- \vee \big(\overrightarrow{\Diamond}\alpha^- \wedge \overrightarrow{\Box}((\overrightarrow{\Diamond}\alpha^- \vee \alpha^-) \rightarrow A)\big) \vee$$
$$\vee \big(\neg\alpha^- \wedge \overrightarrow{\Box}\alpha^- \wedge \overrightarrow{\Diamond}\alpha^+ \wedge \overrightarrow{\Box}((\neg\alpha^- \wedge \overrightarrow{\Box}\alpha^- \wedge \overrightarrow{\Diamond}\alpha^+) \rightarrow$$

as a result, in the tableau system for $\mathcal{L}(MQ+)$, we will only need to use white connectives. We first introduce some preliminary definitions about modal tableaux taken from [8].

A. Preliminary definitions about modal tableaux

Definition 3: A *tableau rule* ρ consists of a *numerator* $\mathcal{N}$ and a finite list of *denominators* $\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_k$ separated by vertical bars:

$$\frac{\mathcal{N}}{\mathcal{D}_1 \mid \mathcal{D}_2 \mid \ldots \mid \mathcal{D}_k}\ (\rho)$$

The numerator of each rule tableau contains one or more distinguished formulae called the *principal formulae*. Each denominator contains one or more distinguished formulae called the *side formulae*.

A *tableau system* is a finite collection of tableau rules $\rho_1, \rho_2, \ldots, \rho_n$. A tableau for a finite set of formulas Γ is a finite tree with root Γ, whose nodes carry finite sets of formulas which have been built by applications of the rules.

[1]This set is usually considered as a subset of the real numbers, but this is not required.

Recall that a tableau rule with numerator $\mathcal{N}$ is applicable to a node containing a set of formulas Δ, precisely if Δ is an instance of $\mathcal{N}$.

We define below a tableau system $T(MQ+)$ for the system $\mathcal{L}(MQ+)$.

1) Rules of $T(MQ+)$: Our tableau method is built in the style of that for linear time logic K4L and adding extra tableau rules for handling frame constants. The proposed tableau system contains the following rules:

- The classical rules from classical propositional logic:

$$\frac{\Gamma; A \wedge B}{\Gamma, A; B}\ (\wedge) \qquad \frac{\Gamma; A; \neg A}{\bot}\ (\bot) \qquad \frac{\Gamma; \neg(A \wedge B)}{\Gamma; \neg A \mid \Gamma; \neg B}\ (\neg\wedge) \qquad \frac{\Gamma; \neg\neg A}{\Gamma; A}\ (\neg) \qquad \frac{\Gamma; \Delta}{\Gamma}\ (\theta)$$

- The modal rules

$$\frac{\Gamma; \neg\overrightarrow{\Box} A}{\Gamma; \overrightarrow{\Diamond}\neg A}\ (\neg\overrightarrow{\Box}) \qquad \frac{\Gamma; \neg\overrightarrow{\Diamond} A}{\Gamma; \overrightarrow{\Box}\neg A}\ (\neg\overrightarrow{\Diamond})$$

- The modal rule $(K4L)$:

$$\frac{\overrightarrow{\Box}\Gamma; \overrightarrow{\Diamond}\Delta}{S_1 \mid S_2 \mid \ldots \mid S_m}\ (K4L)$$

  where $m = 2^n - 1$, and n is the cardinal of Δ. The S_i's are defined as $S_i = (\Gamma; \overrightarrow{\Box}\Gamma; \overrightarrow{\Diamond}\Delta_i{}^c; \Delta_i)$, where $\Delta_1, \ldots, \Delta_m$ is an enumeration of the non-empty subsets of Δ and $\Delta_i{}^c = \Delta \smallsetminus \Delta_i$.
- The following rules, which allow us the handling of frame constants:

$$\frac{\Gamma; \alpha^-}{\Gamma; \overrightarrow{\Box}\neg\alpha^-}\ (uniq^-) \qquad \frac{\Gamma; \alpha^+}{\Gamma; \overrightarrow{\Box}\neg\alpha^+}\ (uniq^+) \qquad \frac{\Gamma; \alpha^-}{\Gamma; \overrightarrow{\Diamond}\alpha^+}\ (ord)$$

Note that in all of these cases, the order of the formulas in the sets is immaterial. Moreover, the static rules of $T(MQ+)$ are the classical rules together with $(\neg\overrightarrow{\Box})$, $(\neg\overrightarrow{\Diamond})$, $(uniq^-)$, $(uniq^+)$ and (ord) whereas $(K4L)$ is the only transitional rule.

Procedure to construct a tableau

1) The root node contains the formulas in Γ. Choose a rule ρ which is applicable to this root node.
2) If ρ has k denominators then create k successor nodes, with successor i carrying an appropriate instance of denominator $\mathcal{D}_i$.
3) Rules are applied non-deterministically to any node which is different from $\{\bot\}$ and whose label has not appeared before in the branch (to avoid loops).
   If after the application of some rule to a node n, a successor s of a node n is labelled with a set Ω which already appeared in the branch from the root to x, then we erase node s and put a link from n to the ancestor which is labelled with Ω.

Definition 4: A branch in a tableau is *closed* if its leaf is $\{\bot\}$, otherwise it is *open*. A tableau is *closed* if all its branches are closed, otherwise it is *open*.

A set Γ is $T(MQ+)$-*consistent* (in the following we will use simply *consistent*) if no tableau for Γ is closed. We say that a formula A is a *theorem* of $T(MQ+)$ if there is a closed tableau for $\{\neg A\}$.

We finish this section with an example of application of the tableaux method.

Example 1: A closed tableau is presented in Fig. 1 which proves the validity of $\alpha^+ \to (\overrightarrow{\Box}\neg\alpha^- \wedge \neg\alpha^-)$

IV. Soundness and Completeness

For soundness all we have to prove is that if the numerator of a given rule is satisfiable, then so is at least one of its denominators. The proof is straightforward. Consider, for example, the rule (ord), and assume that its numerator, that is, $\Gamma; \alpha^-$ is satisfiable, then it is easy to show that $\Gamma; \alpha^-; \overrightarrow{\Diamond}\alpha^+$ is satisfiable as well. The proof for the other rules is similar.

Regarding termination of the tableaux system $T(MQ+)$, it is not difficult to show

Lemma 1: For every finite set Γ we can assign, a priori, a finite set Γ^* such that Γ^* contains all formulae that may appear in any tableau for Γ. As a result, there are only a finite number of tableaux for Γ.

For the proof of completeness we shall show that if Γ is a finite set of formulas for which no tableau closes, then there exists a model for Γ on a frame. Indeed, our method has the following peculiarity: it needs not to guarantee the existence of a model for Γ on a frame for Γ but *on a pre-frame* (depending on the information in Γ about frame constants), because it is always possible to build a frame from the given pre-frame.

The previous comments justify the introduction of the concept of pre-model (that is, a "model" on a pre-frame).

Definition 5: Given a pre-frame Υ, a *multimodal qualitative model on* Υ (or pre-model, for short) is an ordered pair $\mathcal{M} = (\Upsilon, h)$, where h is a *meaning function* (or, *interpretation*) $h\colon \mathcal{V} \longrightarrow 2^{\mathbb{S}}$. Any interpretation can be uniquely extended to the set of all formulas in $\mathcal{L}(MQ+)$ (also denoted by h) by using the usual conditions for the classical boolean connectives and the constants $\top$ and $\bot$, and the following conditions for the modal operators and frame constants:

$$h(\overrightarrow{\Box}A) = \{x \in \mathbb{S} \mid y \in h(A) \text{ for all } y \text{ such that } x < y\}$$

$$h(\alpha^+) = \begin{cases}\{+\alpha\} & \text{if } +\alpha \in \mathbb{S} \\ \varnothing & \text{otherwise}\end{cases} \qquad h(\alpha^-) = \begin{cases}\{-\alpha\} & \text{if } -\alpha \in \mathbb{S} \\ \varnothing & \text{otherwise}\end{cases}$$

Definition 6: A set Γ is *closed with respect to a tableau rule* ρ if whenever an instance of the numerator of the rule is in Γ, so is a corresponding instance of at least one denominator of the rule. A set Γ is *saturated* if Γ is consistent and closed with respect to the static rules of $T(MQ)$ excluding (θ).

The following lemma states that consistent sets can be saturated in an effective way. Its proof is standard and, hence, omitted.

Lemma 2:

1) The rules $(\neg), (\wedge), (\neg\wedge), (uniq^-), (uniq^+)$, and (ord) are *invertible*, that is, satisfy that if there is a closed tableau for (an instance of) the numerator $\mathcal{N}$ then there are closed tableaux for (appropriate instances of) the denominators $\mathcal{D}_i$.

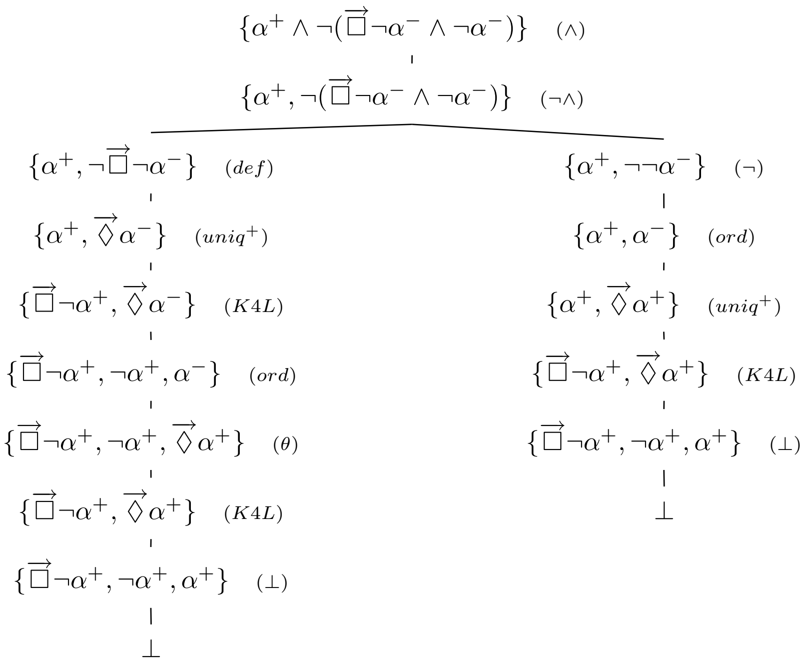


Fig. 1. Closed tableau for Example 1.

2) For each finite consistent set Γ there is an effective procedure to build some finite saturated and consistent set Γ^s being $\Gamma \subseteq \Gamma^s \subseteq \Gamma^*$.

Now, let us recall the definition of *model graph* for a finite set of formulas Γ together with a lemma inspired in [12].

Definition 7: A *model graph* for some finite set of formulas Γ is a multimodal qualitative frame $(\mathbb{S}, +\alpha, -\alpha, <)$ such that all the elements $x \in \mathbb{S}$ are saturated sets satisfying $x \subseteq \Gamma^*$ and

(i) $\Gamma \subseteq x_0$, for some $x_0 \in \mathbb{S}$;
(ii) $\alpha^+ \in +\alpha$ and $\alpha^- \in -\alpha$;
(iii) if $\overrightarrow{\Diamond} A \in x$, then there exists $y \in \mathbb{S}$ such that $x < y$ and $A \in y$;
(iv) if $x < y$ and $\overrightarrow{\Box} A \in x$, then $A \in y$.

Similarly, we define a *pre-model graph* as a multimodal qualitative pre-frame as above, in which condition (ii) is suitably modified (or deleted) depending on which frame constants are missing.

The following lemma ensures the existence of a multimodal qualitative model for Γ on the basis of the existence of a model graph for Γ.

Lemma 3: If Σ is a (pre-)model graph for Γ, then there exists some multimodal qualitative (pre-)model for Γ.

Proof: The definition of h for the atoms and the frame constants in a pre-model is the same than in a model graph; that is,

$$h(p) = \{x \mid p \in x\} \qquad h(\alpha^+) = \{x \mid \alpha^+ \in x\} \qquad h(\alpha^-) = \{x \mid \alpha^- \in x\}$$

By structural induction, we obtain $h(A) = \{x \mid A \in x\}$ for any formula A. QED

The completeness proof will be based on the following technical result

Proposition 1: Consider a pre-model graph Ψ with an initial point. Let Γ be a set of formulas with no occurrence of either α^+ or α^-. If there exists a pre-model (Ψ, h) of Γ, then there exists a model (Ψ', h') of Γ.

Proof: To begin with, let us introduce some specific saturated sets containing the frame constants:

$$x_{\alpha^-} = \{\alpha^-, \overrightarrow{\Diamond}\alpha^+, \overrightarrow{\Box}\neg\alpha^-\}$$
$$x_{\alpha^+} = \{\alpha^+, \neg\alpha^-, \overrightarrow{\Box}\neg\alpha^-, \overrightarrow{\Box}\neg\alpha^+\}$$

The intuition underlying this is that x_{α^-} represents $-\alpha$ and x_{α^+} represents $+\alpha$ in a frame.

Depending on what frame constants fail to appear in the pre-frame, just two possibilities arise (note that it is not possible that α^- appears in some set and α^+ does not appear in any set in the sequence of points of the pre-frame):

1) If α^- does not appear in any set in the sequence, then the set x_{α^-} is prepended to the sequence.
2) If neither α^- nor α^+ appear in any set in the sequence, then the sets x_{α^-} and x_{α^+} are prepended to the sequence in that order.

Once the frame constants have been introduced in the sequence, some post-processing is needed in order to preserve coherence. This is done as a consequence of the following results:

- Given condition (1) above, if x is a saturated set of the sequence, then the set $x \cup \{\neg\alpha^-, \overrightarrow{\Box}\neg\alpha^-\}$ is saturated.
- Given condition (2) above, if x is a saturated set of the sequence, then the set $x \cup \{\neg\alpha^-, \overrightarrow{\Box}\neg\alpha^-, \neg\alpha^+, \overrightarrow{\Box}\neg\alpha^+\}$ is saturated.

In order to prove the first result above, let us consider the following embedding from Ψ to Ψ' defined by $x \mapsto$

$x \cup \{\neg\alpha^-, \overrightarrow{\Box}\neg\alpha^-\}$.

With respect the second result above, let us consider the following embedding from Ψ to Ψ' defined by $x \mapsto x \cup \{\neg\alpha^-, \overrightarrow{\Box}\neg\alpha^-, \neg\alpha^+, \overrightarrow{\Box}\neg\alpha^+\}$.

Then, in any case, by an induction argument, it is possible to prove that $h(A) = h'(A)$ for all formula $A \in \Gamma$. QED

We will work with a generalization of the idea of model graph which permits clusters among the saturated sets, and we will simply consider a *graph* as an ordered pair (X, R) where X is a non-empty set of saturated sets and R is a transitive relation on X. This way, a model graph a just a particular instance of this more general concept. Before stating the theorem, the notion of fulfilled eventuality has to be introduced.

Definition 8: Let (X, R) be a graph. Take a point $x \in X$, we say that x *fulfills an eventuality* $\overrightarrow{\Diamond} A$ if $x \in h(A)$. A sequence of points $x_1 R x_2 R x_3 R \ldots$ is said to fulfill an eventuality $\overrightarrow{\Diamond} A$ if there are points x_i, x_j in the sequence (with $x_i R x_j$) such that $\overrightarrow{\Diamond} A \in x_i$ and $A \in x_j$.

Theorem 1 (Completeness): If Γ is a finite consistent set of formulas, then there is a model for Γ on a finite multimodal qualitative frame $(\mathbb{S}, +\alpha, -\alpha, <)$.

Proof: By Lemma 3 it is sufficient to build a model graph for Γ. This model graph will be constructed step by step by successively fulfilling eventualities, which means that saturated sets are being added to an initial graph in a way guided by the tableau rules.

There are several novelties in this construction with respect to other well-known standard methods:

1) First of all, the consideration of frame constants requires a careful procedure to ensure that they will eventually be constructed in the model.
2) Moreover, rules do not always provide the constants α^+, α^- in the sequence of saturated sets that will be constructed. However, this is not a handicap, for in that case we would obtain a pre-frame satisfying Γ which can be extended to a frame which still is a model of Γ.
3) Finally, it is worth mentioning that the generated model needs not be always finite; nevertheless, the application of the rules always terminate. More comments on this will be given when the formal procedure has been introduced.

STEP 1. Our first task is to obtain a graph in which every eventuality is fulfilled, mark that such a graph can contain clusters. For this end, we can follow a pattern similar to that [8] for $\mathcal{CS}4.3$. Once the construction is terminated, clusters can be eliminated by applying *bulldozing* techniques.

STEP 2. If the graph constructed so far turns out to be a frame, then by Lemma 3 we obtain that Γ has a model and the method stops. Otherwise, we go to Step 3.

STEP 3. In this case, we have just a pre-frame, however a model based on a frame extending the pre-frame can be conveniently constructed by Proposition 1. QED

V. CONCLUSIONS AND FUTURE WORK

The use of a logical apparatus in the treatment of qualitative reasoning has allowed the development of a tableau method for testing satisfiability in $\mathcal{L}(MQ+)$. Although the logic $\mathcal{L}(MQ)$ has just two landmarks, and is considerably simpler than those stated at the beginning of this section, still it is useful as a stepping stone for considering more complex systems, for which the logic has to be enriched by adding new modal operators capable to treat a bigger number of milestones, equivalence classes and/or qualitative relations.

As future work in this context, it is planned to extend the tableau method for the full logic $\mathcal{L}(MQ)$ with past and future operators, as well as to investigate tableau systems for the extended language with negligibility relations of [5]. Last but not least, we are also investigating the feasibility of providing a Rasiowa-Sikorsi proof system for a relational presentation of the logic.

REFERENCES

[1] B. Bennett. Modal logics for qualitative spatial reasoning. *Bull. of the IGPL*, 3:1–22, 1995.

[2] B. Bennett, A.G. Cohn, F. Wolter, and M. Zakharyaschev. Multi-Dimensional Modal Logic as a Framework for Spatio-Temporal Reasoning. *Applied Intelligence*, 17(3): 239–251, 2002.

[3] J. P. Burgess. Basic tense logic. In D. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic: Volume II: Extensions of Classical Logic*, pages 89–133. Reidel, Dordrecht, 1984.

[4] A. Burrieza and M. Ojeda-Aciego. A multimodal logic approach to order of magnitude qualitative reasoning. *Lect. Notes in Artificial Intelligence* 3040, pages 431-440, 2004.

[5] A. Burrieza and M. Ojeda-Aciego. A multimodal logic approach to order of magnitude qualitative reasoning with comparability and negligibility relations. *Fundamenta Informaticae*, 2005. To appear.

[6] P. Dague. Numeric reasoning with relative orders of magnitude. In *Proc. 11th National Conference on Artificial Intelligence*, pages 541–547. The AAAI Press/The MIT Press, 1993.

[7] P. Dague. Symbolic reasoning with relative orders of magnitude. In *Proc. 13th Intl. Joint Conference on Artificial Intelligence*, pages 1509–1515. Morgan Kaufmann, 1993.

[8] R. Goré. Tableau methods for modal and temporal logics. In M. D'Agostino and D. Gabbay and R. Hähnle and J. Posegga, editors, *Handbook of Tableau Methods*, pages 297–396. Kluwer, Dordrecht, 1999.

[9] M.L. Mavrovouniotis and G. Stephanopoulos. Reasoning with orders of magnitude and approximate relations. In *Proc. 6th National Conference on Artificial Intelligence*. The AAAI Press/The MIT Press, 1987.

[10] O. Raiman. Order of magnitude reasoning. *Artificial Intelligence*, 51:11–38, 1991.

[11] D. Randell, Z. Cui, and A. Cohn. A spatial logic based on regions and connections. *Proc. of the 3rd Intl Conf on Principles of Knowledge Representation and Reasoning* (KR'92), pg 165–176, 1992.

[12] W. Rautenberg. Modal tableau calculi and interpolation. *Journal of Philosophical Logic* 12:403–423, 1983.

[13] B. Shults and B.J. Kuipers. Proving properties of continuous systems: qualitative simulation and temporal logic. *Artificial Intelligence*, 92:91–129, 1997.

[14] M. Sánchez, F. Prats, and N. Piera. A formalization of comparability relations in qualitative models [in Spanish]. *Bulletin of the Spanish Association for AI*, 6:15–22, 1996.

[15] L. Travé-Massuyès, F. Prats, M. Sánchez, and N. Agell. Consistent relative and absolute order-of-magnitude models. In *Proc. Qualitative Reasoning 2002 Conference*, 2002.

[16] F. Wolter and M. Zakharyaschev. Qualitative spatio-temporal representation and reasoning: a computational perspective. In G. Lakemeyer and B. Nebel, editors, *Exploring Artificial Intelligence in the New Millenium*. Morgan Kaufmann, 2002.